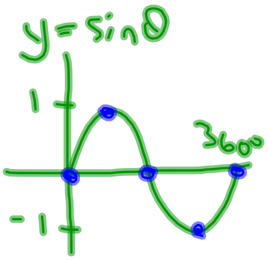
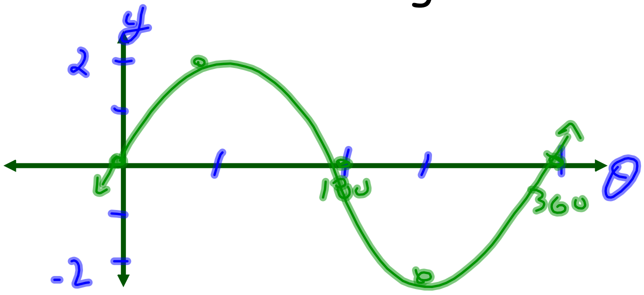
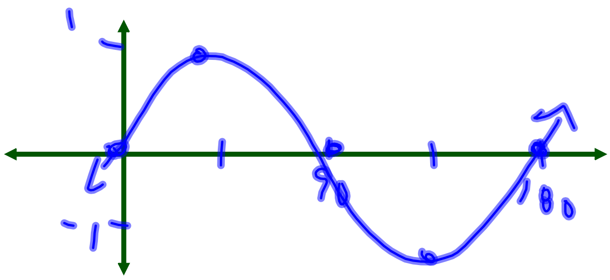
EXAMPLES: Sketch each of the following...

a) $y = 2\sin\theta$

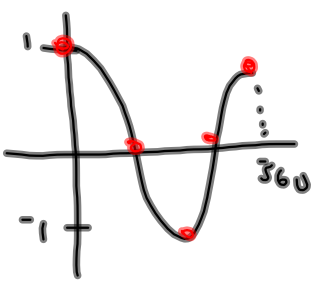


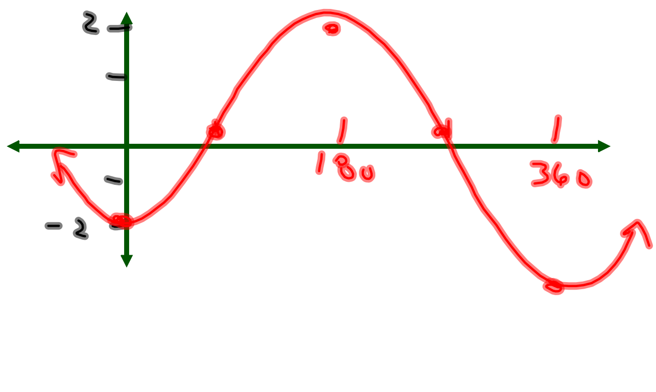


b) $y = \sin(2\theta)$



c) $y = -2\cos\theta$





Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $f(\theta) = a\sin[k(\theta - c)] + d$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | .

3. Period: The period of the graph will be equal to $\frac{360°}{k}$

4. Horizontal Phase Shift: The graph will shift "c" units to the right. (Think Opposite)

5. Vertical Translation: The graph will shift "d" units up.

Mapping Notation: $(x, y) \rightarrow \left(\frac{1}{k}\theta + c, ay + d\right)$

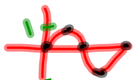
Transformations of Sinusoidal Functions

Example: $f(\theta) = -2\sin 3(\theta + 30°) - 2$

Domain	
Range	
Reflection	
Amplitude	
Horizontal Phase Shift	
Vertical Translation	
Period	

EXAMPLE #1

Now let's sketch a graph of $f(\theta) = -2\sin 3(\theta + 30°) - 2$

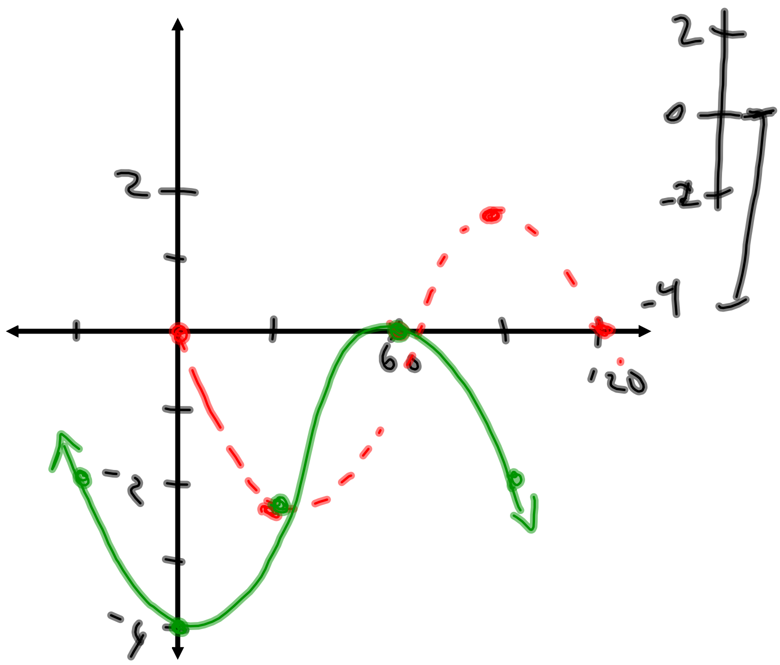


" THINK: RST "

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation sec*

DOMAIN	$\theta \in R$
RANGE	$-4 \leq y \leq 0$
AMPLITUDE	2
PERIOD	$\frac{360°}{3} = 120°$
PHASE SHIFT	30° Left
VERTICAL TRANSLATION	Down 2
EQUATION OF SINUSOIDAL AXIS	$y = -2$

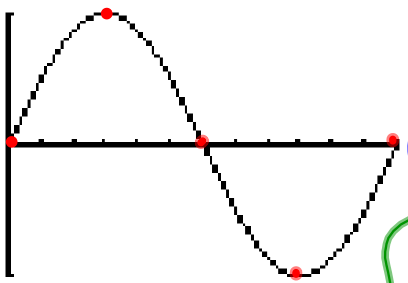


Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$f(\theta) = -2\sin 3(\theta + 30^\circ) - 2$$

$y = \sin\theta$



Mapping:

$(\theta, y) \rightarrow \left(\frac{1}{3}\theta - 30, -2y - 2\right)$

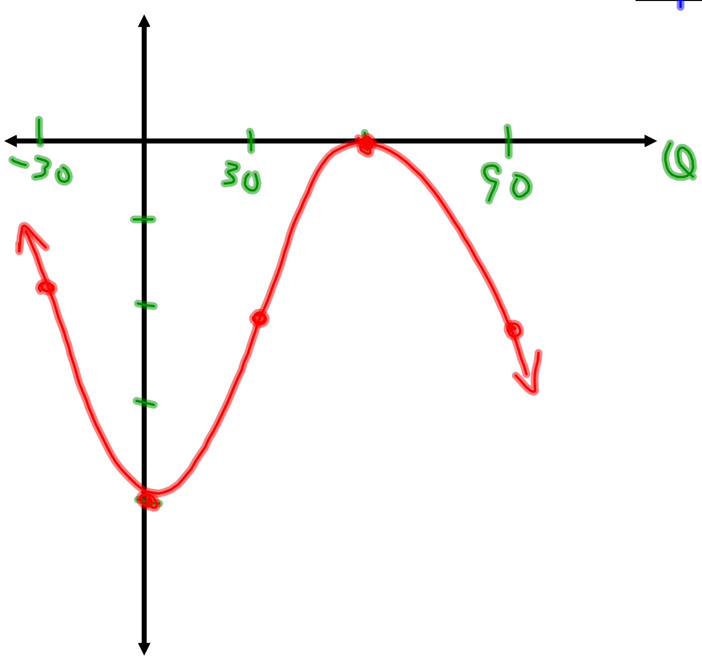
$y = \sin\theta$

$(0,0) \rightarrow \left(\frac{1}{3}(0) - 30, -2(0) - 2\right)$

θ	y
0	0
90	1
180	0
270	-1
360	0

New points after mapping

θ	y
-30	-2
0	-4
30	-2
60	0
90	-2



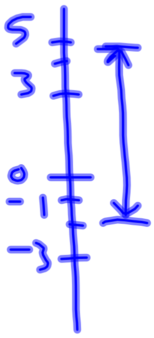
EXAMPLE #2

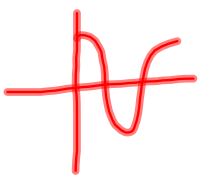
Now let's sketch a graph of $y = 3\cos\left[2\left(\theta - 135^\circ\right)\right] + 2$

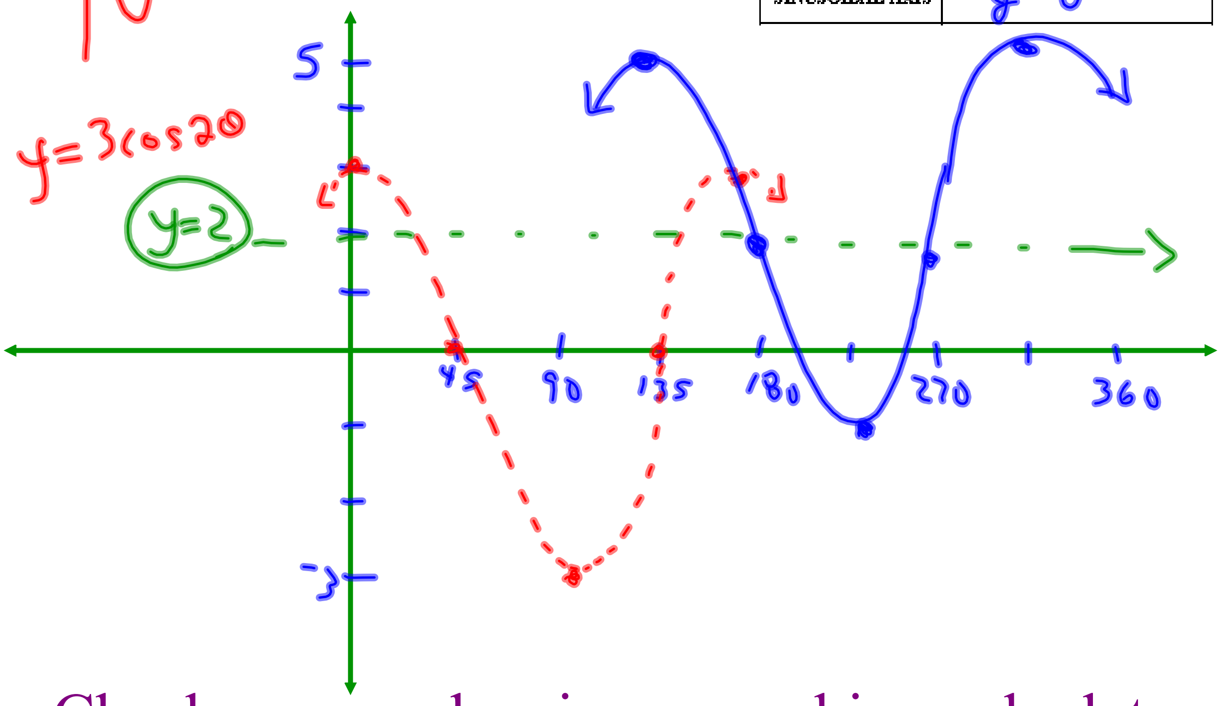
Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

DOMAIN	$\theta \in R$
RANGE	$-1 \le y \le 5$
AMPLITUDE	3
PERIOD	180°
PHASE SHIFT	Right 135°
VERTICAL TRANSLATION	Up 2
EQUATION OF SINUSOIDAL AXIS	$y = 2$



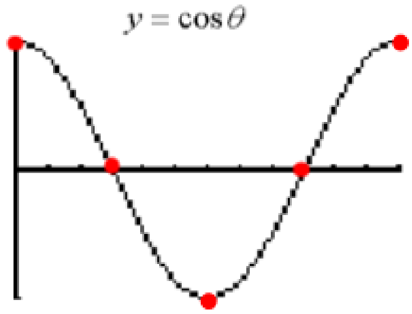




Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$y = 3\cos[2(\theta - 135°)] + 2$$



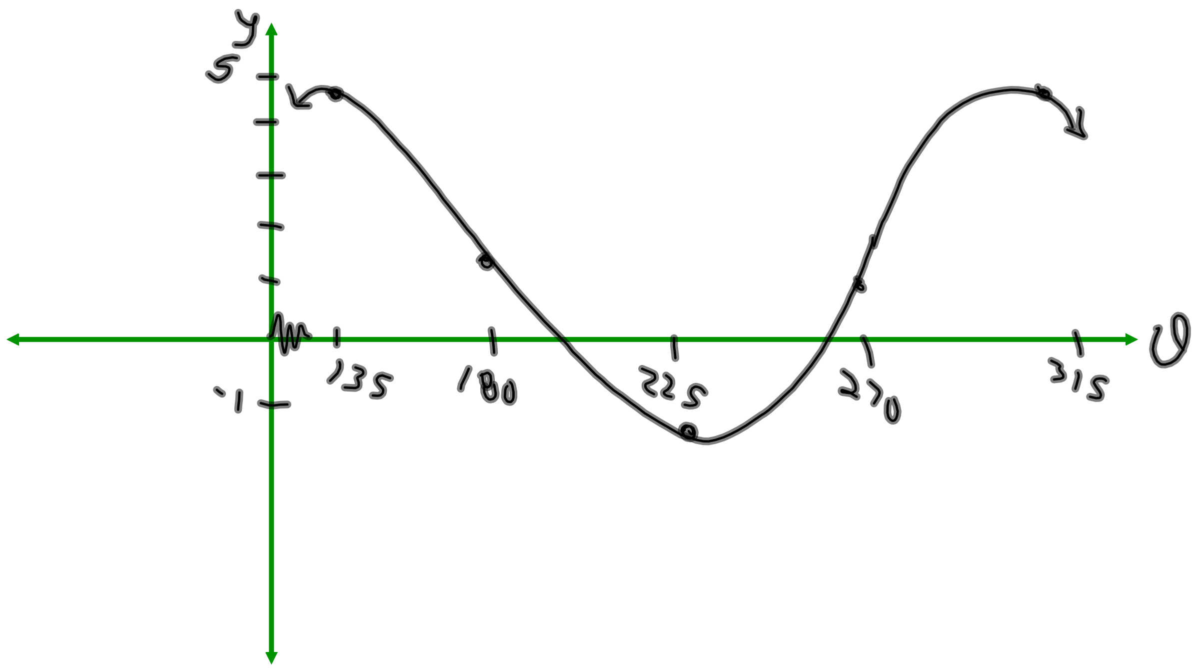
Mapping:

$(\theta, y) \rightarrow \left(\frac{1}{2}\theta + 135°, 3y + 2\right)$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
135	5
180	2
225	-1
270	2
315	5



Hopefully you are not too puzzled for this one...

$$(2)\quad \frac{1}{\cancel{2}}(y+1)=\overset{(2)}{3}\cos\left(\frac{1}{2}\theta-90^\circ\right)+2\ (2)$$

Remember...Put in standard form first!!

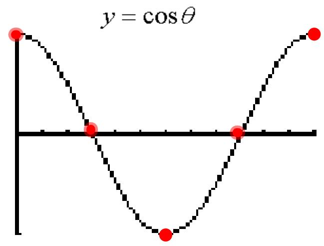
$$y+1 \overset{-1}{=} 6\cos\left(\tfrac{1}{2}\theta-90^\circ\right)+4 \quad -1$$

$$y=6\cos\left(\tfrac{1}{2}(\theta-180^\circ)\right)+3$$

$$2y=2(5\cos 7x)$$

$$y=10\cos 7x$$

Remember what the graph of cosine looks like ??



Mapping:

θ	y
0	
90	
180	
270	
360	

New points after mapping

θ	y

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS