Logarithmic Functions

Did You Know?

Logarithms were developed independently by John Napier (1550–1617), from Scotland, and Jobst Bürgi (1552–1632), from Switzerland. Since Napier published his work first, he is given the credit. Napier was also the first to use the decimal point in its modern context.

Logarithms were developed before exponents were used. It was not until the end of the seventeenth century that mathematicians recognized that logarithms are exponents.

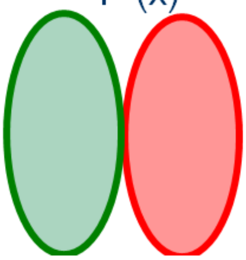
Recall some knowledge about inverse functions...

•Inverse functions is the set of ordered pair obtained by interchanging the x and y values.

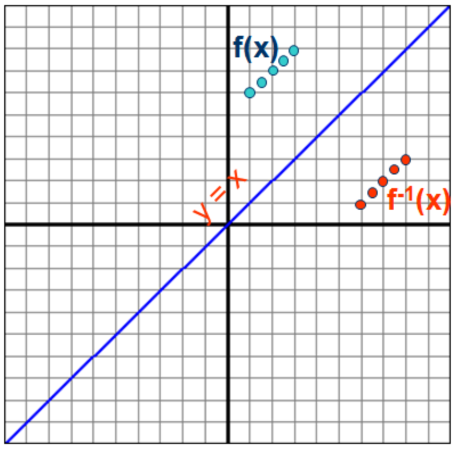
f(x)

x	y
2	12
3	13
4	14
5	15
6	16

$f^{-1}(x)$



•Inverse functions can be created graphically by a reflection on the y = x axis.



Word association game....

Addition ... inverse process ...??

Multiplication ... inverse process ...??

Squaring ... inverse process ...??

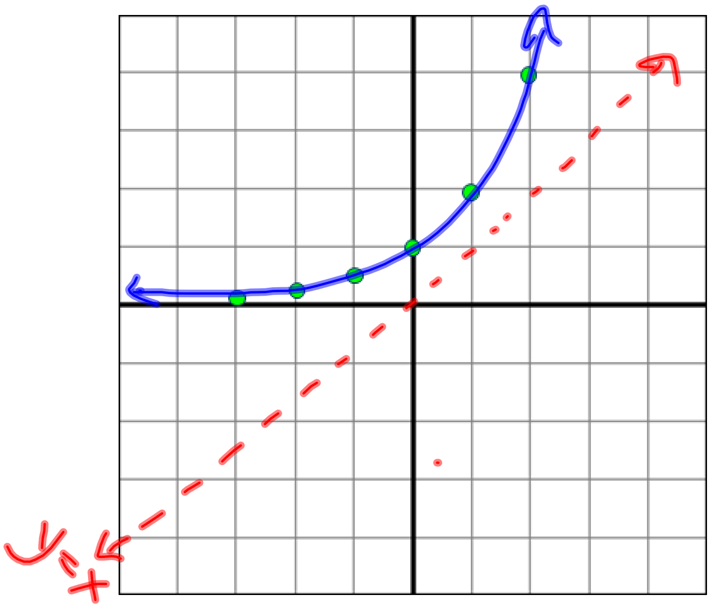
Sine ... inverse process ...??

Exponential ... inverse process ...??

•Let us graph the exponential function $y = 2^x$

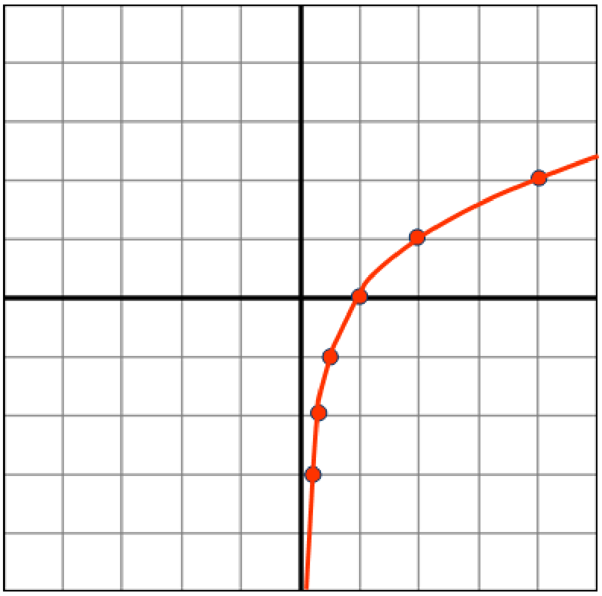
•Table of values:

x	y
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4



- Let us find the inverse the exponential function $y = 2^x$
- Table of values:

x	y
0.125	-3
0.25	-2
0.5	-1
1	0
2	1
4	2



•Next, you will find the inverse of an exponential algebraically
•Write the process in your notes

$y = a^x$

Interchange $x \rightarrow y$

$x = a^y$

•We write these functions as:

$x = a^y \longrightarrow y = \log_a x$

base

$$x = a^y$$

exponent

$$y = \log_a x$$

exponent base

IMPORTANT NOTATION!!!

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression $\log_6 1$, the argument is 1.

Exponential Form		**Logarithmic Form**	
$x = a^y$	is written as…	$y = \log_a x$	← read as… "log of x to the base a"
Say, "the base a to the exponent y is x."		Say, "y is the exponent to which you raise the base a to get the answer x."	$a > 0$ and $a \neq 1$

Example 1) Write the following into logarithmic form:

a) $3^3 = 27$ → $\log_3 27=3$

b) $4^5 = 256$ → $\log_4 256=5$

c) $2^7 = 128$ → $\log_2 128=7$

d) $(1/3)^x=27$ → $\log_{1/3} 27=x$

Example 2) Write the following into exponential form:

a) $\log_2 64=6$ ⟶ $2^6 = 64$

b) $\log_{25} 5=1/2$ ⟶ $25^{1/2} = 5$

c) $\log_8 1=0$ ⟶ $8^0 = 1$

d) $\log_{1/3} 9=2$ ⟶ $(1/3)^2 = 1/9$

Example 3) Find the value of x for each example:

a) $\log_{1/3}27 = x$

$\frac{1}{3}^{x} = 27$

$3^{-x} = 3^{3}$

$-x = 3$

$x = -3$

$\left(\frac{1}{3}\right)^{-3} = 3^{3}$

$= 27$

b) $\log_5 x = 3$

$5^3 = x$

$125 = x$

c) $\log_x(1/9) = 2$

$\sqrt{x^2} = \sqrt{\frac{1}{9}}$

$x = \frac{1}{3}$

d) $\log_3 x = 0$

$3^0 = x$

$1 = x$

$\log_5 30 \Rightarrow$ "5 to what exponent gives 30"

$5^2 = 25$

$5^3 = 125$ ≈ 2.2

$0.125 = \frac{1}{2}^x$

" $\frac{1}{2}$ Raised to some exponent will equal 0.125 "

$\log_{\frac{1}{2}} 0.125$

1x What is $\log_{10} 1000$?

Since our number system is based on powers of 10, **logarithms** with base 10 are widely used and are called **common logarithms**. When you write a common logarithm, you do not need to write the base. For example, $\log 3$ means $\log_{10} 3$.

So now after reading the above...What is log 1000 ?

$= 3$

Review...

logarithmic function

- a function of the form $y = \log_c x$, where $c > 0$ and $c \neq 1$, that is the inverse of the exponential function $y = c^x$

logarithm

- an exponent
- in $x = c^y$, y is called the logarithm to base c of x

common logarithm

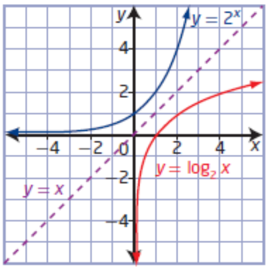
- a logarithm with base 10

Key Ideas

- A logarithm is an exponent.
- Equations in exponential form can be written in logarithmic form and vice versa.

Exponential Form	**Logarithmic Form**
$x = c^y$	$y = \log_c x$

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.
- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown.



- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,
  - the domain is $\{x \mid x > 0, x \in R\}$
  - the range is $\{y \mid y \in R\}$
  - the x-intercept is 1
  - the vertical asymptote is $x = 0$, or the y-axis
- A common logarithm has base 10. It is not necessary to write the base for common logarithms:

$$\log_{10} x = \log x$$

Check-Up Time...

Evaluate each of the following:

1. $\log_2 8\sqrt{32} = x$

$\log_2 2^3(2^5)^{1/2}$

$\log_2 2^3(2^{5/2})$

$\log_2 2^{11/2} = x$

$= \frac{11}{2}$

$(x^2)^{1/2} = 9$

2. $-\frac{2}{3} = \log_x 81$

$(x^{-2/3})^{-3/2} = (81)^{-3/2}$

$x^1 = \frac{1}{81^{3/2}} = \frac{1}{729}$

3. $\log_5 \frac{1}{125} = x$

$5^x = \frac{1}{125}$

$5^x = \frac{1}{5^3}$

$5^x = 5^{-3}$

$x = -3$

4. $\log_{\sqrt{6}} 36 = x$

$(\sqrt{6})^x = 36$

$6^{\frac{1}{2}x} = 6^2$

$\frac{1}{2}x = 2$

$x = 4$

What is the value of any logarithm with an argument of 1? Why?

General Properties of Logarithms:

If $a > 0$ and $a \neq 1$, then…

(i) $\log_a 1 = 0$

(ii) $\log_a a^x = x$

(iii) $a^{\log_a x} = x$