Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

$\log_2 (3^2)^2 + \log_2 6 - \log_2 3^3$

$\log_2 3^4 + \log_2 6 - \log_2 3^3$

$\log_2\left(\frac{3^4 \cdot 6}{3^3}\right) = \log_2 18$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$

$\frac{1}{3}\log_2 32$

$\frac{1}{3}\log_2 2^5$

$\frac{1}{3}(5) = \frac{5}{3}$

c) Express the following as a single logarithm... $\frac{1}{2}[(\log_5 a + 2\log_5 b) - 3\log_5 c]$

$\log_5 \frac{\sqrt{a}\, b}{\sqrt{c^3}}$ ✓

$\log_5 \frac{a^{1/2} b}{c^{3/2}}$

$\frac{1}{2}(\log_5 a + \log_5 b^2 - \log_5 c^3)$

✓ $\frac{1}{2}\left(\log_5 \frac{ab^2}{c^3}\right)$

✓ $\log_5 \left(\frac{ab^2}{c^3}\right)^{1/2} = \log_5 \sqrt{\frac{ab^2}{c^3}}$

d) Express as a single logarithm in simplest form...

$$\frac{3}{4}\left[12(\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$$

$\frac{3}{4}$

$\frac{3}{4}(12\log_b x^2 - 24\log_b x + 8\log_b x^{1/2} - 4\log_b x^{-7})$

① Remove Brackets

$9\log_b x^2 - 18\log_b x + 6\log_b x^{1/2} - 3\log_b x^{-7}$

② Return all exponents!!

$\log_b (x^2)^9 - \log_b x^{18} + \log_b (x^{1/2})^6 - \log_b x^{-21}$

$\cancel{\log_b x^{18}} - \cancel{\log_b x^{18}} + \log_b x^3 - \log_b x^{-21}$

$\log_b \left(\frac{x^3}{x^{-21}}\right)$

or $= \log_b x^{24}$

$= 24\log_b x$

$\frac{3}{4}\log_b x^{32}$

$\log_b x^{32(\frac{3}{4})}$

$\log_b x^{24}$

$\log_x w = -4 \qquad \log_x r = 3 \qquad \log_x a = 5$

① Evaluate: $\log_x\left(\frac{w^3 r}{\sqrt{a}}\right)$

$\log_x w^3 + \log_x r - \log_x a^{1/2}$

$3\log_x w + \log_x r - \frac{1}{2}\log_x a$

$3(-4) + 3 - \frac{1}{2}(5)$

$-\frac{9}{1} - \frac{5}{2}$

$-\frac{23}{2}$

$x^{-4} = w \qquad x^3 = r$

$x^5 = a$

$\log_x\left(\frac{(x^{-4})^3 x^3}{(x^5)^{1/2}}\right)$

$\log_x\left(\frac{x^{-12} x^3}{x^{5/2}}\right)$

$\log_x x^{-23/2}$

$= -\frac{23}{2}$

② $\log_x \sqrt[3]{\frac{a^5 r^2}{x^7 w^{-3}}}$

$\log_x\left(\frac{a^{5/3} r^{2/3}}{x^{7/3} w^{-1}}\right)$

$-\log_x w^{-1}$

$\frac{5}{3}\log_x a + \frac{2}{3}\log_x r - \frac{7}{3}\log_x x + \log_x w$

$\frac{5}{3}(5) + \frac{2}{3}(3) - \frac{7}{3}(1) + (-4)$

$\log_x x = 1$

$\frac{25}{3} + \frac{6}{3} - \frac{7}{3} - \frac{12}{3}$

$\frac{12}{3}$

$= 4$

$$\log_3\left[\log_x\left(\log_3 9\sqrt{27}\right)\right] = -3$$

$$\log_3 3^2(3^3)^{1/2}$$

$$\log_3\left(3^2 \cdot 3^{3/2}\right)$$

$$\log_3 3^{7/2}$$

$$= \frac{7}{2}$$

$$\log_3\left[\log_x\left(\frac{7}{2}\right)\right] = -3 \qquad \log_3 M = -3$$

$$3^{-3} = \log_x \frac{7}{2}$$

$$\frac{1}{27} = \log_x \frac{7}{2}$$

$$\left(x^{\frac{1}{27}}\right)^{27} = \left(\frac{7}{2}\right)^{27}$$

$$x = \left(\frac{7}{2}\right)^{27}$$

Solving Logarithmic Equations

STEPS... (1) Write left side & right side as a single logarithm

NOTE: $\log_a a = 1$

(2) Set arguments equal & solve the equation

(3) Check for extraneous roots

$\log_x(R) = \log_x(W)$

$R = W$

EXAMPLE #1: $\log_3 x - \log_3 4 = \log_3 12$

$\log_3\left(\frac{x}{4}\right) = \log_3 12$

$\frac{x}{4} = 12$

$x = 48$ ✓

EXAMPLE #2:

Solve the following equation... $\log_{10}(x+2) + \log_{10}(x-1) = 1$

-4

$\log_{10}[(x+2)(x-1)] = 1$

Option 1

$10^1 = (x+2)(x-1)$

$10 = x^2 + x - 2$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$\cancel{x = -4}$ or $\boxed{x = 3}$

extraneous

Option 2:

$\log_{10}[(x+2)(x-1)] = \log_{10} 10^1$

$(x+2)(x-1) = 10$

EXAMPLE #3:

$(x+5) \qquad (x+4)$

Solve: $\log_2(x+1) + \log_2(x-1) = 3$ $\quad x = 3$

$\log_2(x+1)(x-1) = 3$

$2^3 = (x+1)(x-1)$

$8 = x^2 - 1$

$0 = x^2 - 9$

$0 = (x-3)(x+3)$

✓ $\boxed{x = 3}$ or $\cancel{-3}$

extraneous

Exponential Equations

What if both sides can not be written to powers of a common base?

Example: $3^x = 30$

What would this equation be if expressed as a logarithmic statement?

$\log_3 30 = x$ Can this be determined using a calculator?

Here is a new method to solve exponential equations...

- Particularly effective when unable to express both sides as a power of a common base

Key property of equations...

$e = 2.71...$

- As long as you perform the same operation to BOTH sides of an equation, equality will be maintained

Take the common logarithm of both sides...or natural logarithm

Why base 10 or base "e" ?

$$3^x = 30$$

$$\log_? 3^x = \log_? 30$$

$$\frac{x(\log 3)}{\log 3} = \frac{\log 30}{\log 3}$$

$$x \doteq 3.096$$

$$\frac{6x}{6} = \frac{17}{6}$$

$$x = \frac{17}{6}$$

$$\ln 3^x = \ln 30$$

$$x \ln 3 = \ln 30$$

$$x = \frac{\ln 30}{\ln 3}$$

Example: $6^{2x-3} = 8^{x+1}$

$$\log 6^{2x-3} = \log 8^{x+1}$$

$$(2x-3)\log 6 = (x+1)\log 8$$

$$2x\log 6 - 3\log 6 = x\log 8 + \log 8$$

$$2x\log 6 - x\log 8 = \log 8 + 3\log 6$$

$$\frac{x\cancel{(2\log 6 - \log 8)}}{\cancel{2\log 6 - \log 8}} = \frac{(\log 8 + 3\log 6)}{(2\log 6 - \log 8)}$$

$$\underline{x = 4.956}$$

$5x - 3 = 3x + 7$

$5x - 3x = 7 + 3$

4.396

-2.795

Example: $\log\left(\dfrac{2^{4x}}{5^{2x+5}}\right) = \log(6^{x-1})$

$$\log 2^{4x} - \log 5^{2x+5} = \log 6^{x-1}$$

$$4x\log 2 - (2x+5)\log 5 = (x-1)\log 6 \rightarrow (\log 6)x$$

$$4x\log 2 - 2x\log 5 - 5\log 5 = x\log 6 - \log 6$$

$$x(4\log 2 - 2\log 5 - \log 6) = -\log 6 + 5\log 5$$

$$x = \frac{(-\log 6 + 5\log 5)}{(4\log 2 - 2\log 5 - \log 6)}$$

$$\frac{\cancel{10}x}{\cancel{10}} = \frac{37}{10}$$

$$\underline{x = -2.795}$$