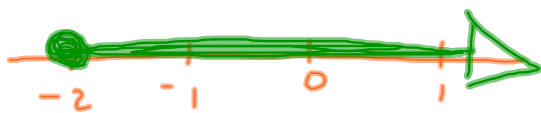# Bracket Notation

$x \geq -2$

$[-2, \infty)$

$(-1, 2]$
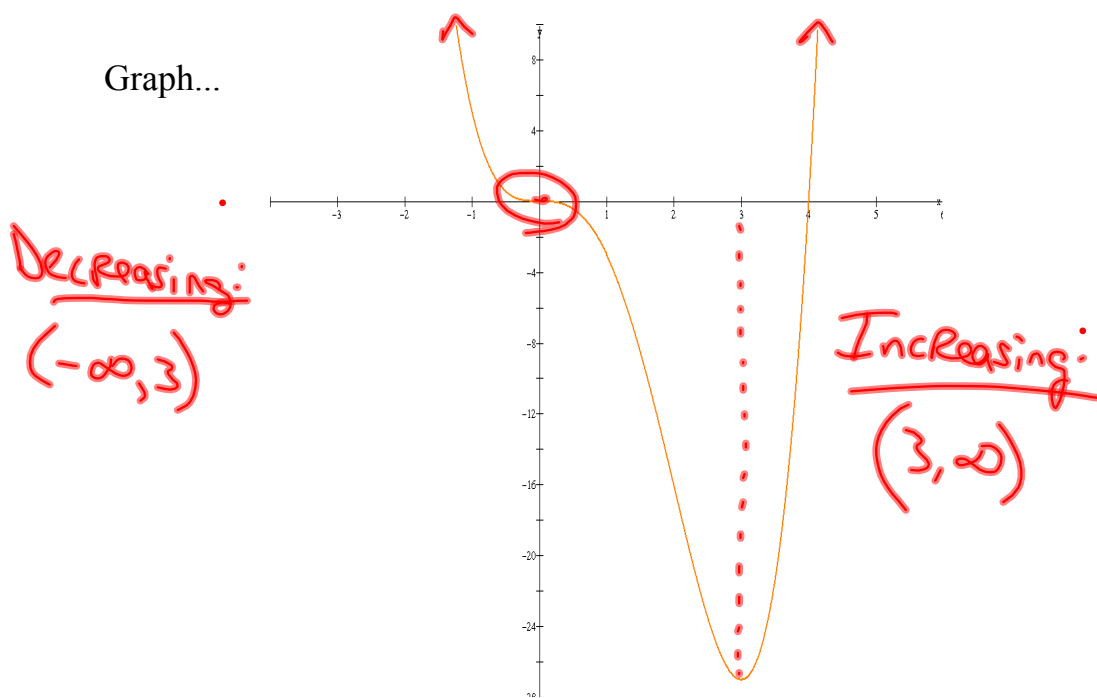

$(-\infty, 3)$

$(2, 3)$

Intervals of Increase and Decrease...

Given the function $f(x) = x^4 - 4x^3$, use the graph below to determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing.

Graph...

Decreasing:

$(-\infty, 3)$

Increasing:

$(3, \infty)$

The Calculus of Intervals of Increase and Decrease

- Examine this graph for intervals of increase and decrease...

Intervals of increase?

(B, c)

"m"
is
y'

Intervals of decrease?

$(-\infty, B)$ (C, ∞)
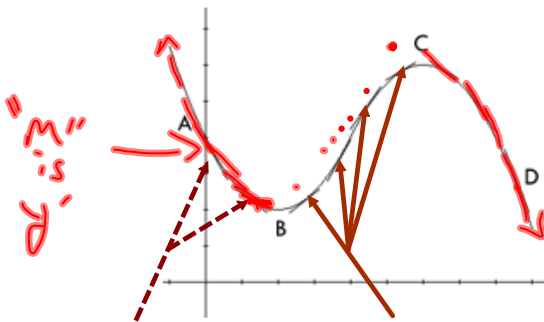
What do you notice about the slopes of these tangents?

What do you notice about the slopes of these tangents?

Critical Value(s):

Any value of x such that $f'(x) = 0$ or $f'(x)$ is undefined.

**Where does _f(x)_ switch from increasing to decreasing? How would this tie in with Calculus?**

- _At the point where a function switches from increasing to decreasing, or decreasing to increasing, the derivative must be equal to 0 or undefined._
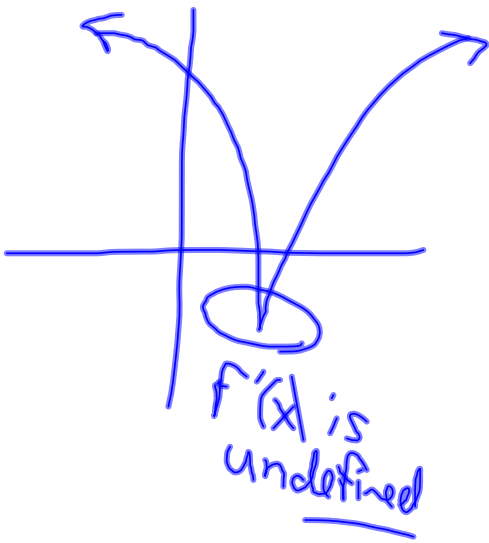
Let's summarize how Calculus could be used to identify regions of increase or decrease...

If $f'(x) > 0$ ----→ f(x) is increasing

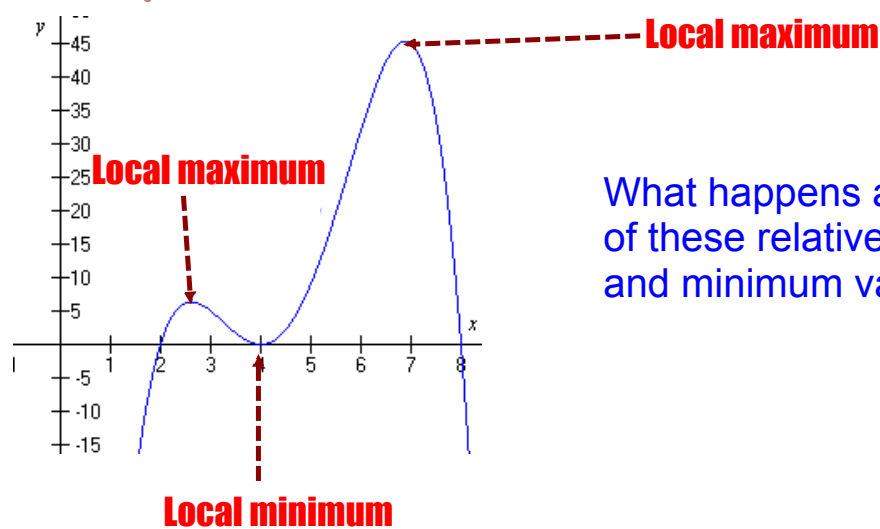If $f'(x) < 0$ ----→ f(x) is decreasing

The graphs below illustrate the first derivative test.

$f'(x)>0$ $f'(x)<0$ $f'(x)<0$ $f'(x)>0$ $f'(x)>0$ $f'(x)>0$ $f'(x)<0$ $f'(x)<0$

A   B   C   D

$f'(x)$ is
<u>undefined</u>
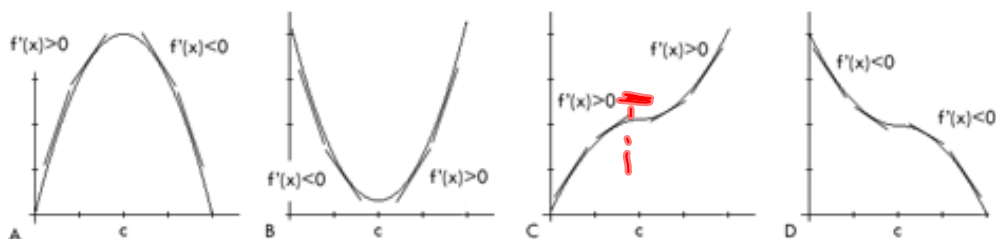
Relative Extrema: (local maximum or local minimum)

Local maximum

Local maximum

Local minimum

What happens at each of these relative maximum and minimum values?

We say that $f(x)$ has a **relative (or local) maximum** at $x = c$ if $f(x) \leq f(c)$ for every x in some open interval around $x = c$.
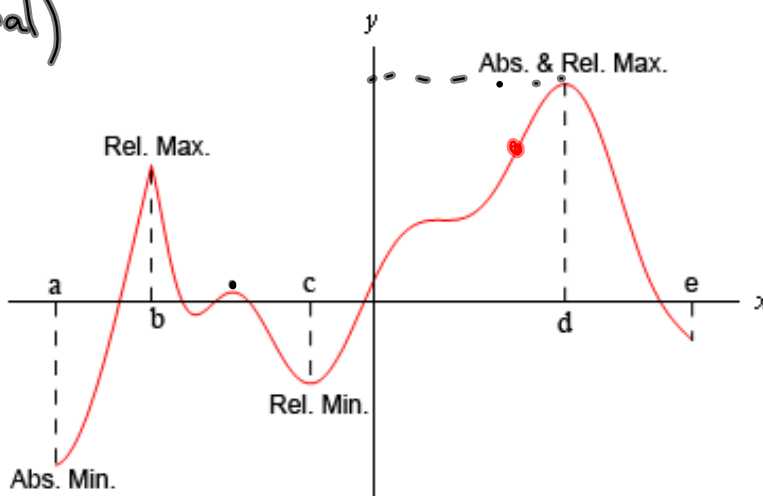
- function switches from increasing to decreasing.

We say that $f(x)$ has a **relative (or local) minimum** at $x = c$ if $f(x) \geq f(c)$ for every x in some open interval around $x = c$.

- function switches from decreasing to increasing.

Absolute Maximum: (Global)

(Global) Absolute Minimum:

Rel. Max.

Abs. & Rel. Max.

Rel. Min.

Abs. Min.

Intervals of increase and decrease can be
organized using a first derivative sign table...

Example:

Determine the intervals of increase and decrease for the function ...

$$f(x) = x^4 - 4x^3 + 2 \quad, -12 \leq x \leq 6$$

1) Critical Values:

$$f'(x) = 4x^3 - 12x^2$$

Factor $f'(x)$:

$$0 = 4x^2 (x-3)$$

Critical
Values : $x = 0, 3$

$$\frac{+}{3}$$

Local Maximum

None

Sign Table:

Interval	$4x^2$	$x-3$	f'	f
$(-\infty, 0)$	+	−	−	Dec
$(0, 3)$	+	−	−	Dec
$(3, \infty)$	+	+	+	Inc

Factors of f'

Local Minimum

$$\left(3, -25\right) \quad f(3) = -25$$

6

Another example...

6. **(15 pts.)** Consider $f(x) = 6x^2 - x^3$.

    a. **(7 pts.)** Identify the intervals on which $f(x)$ is increasing and decreasing. Give the ordered pair (x, y) at any local extrema, identifying each as a local maximum or local minimum. Put your answers in the blanks below.
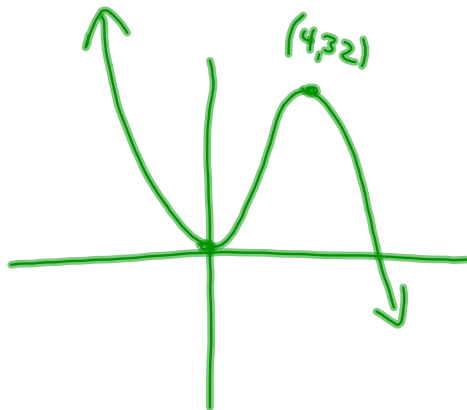
$f'(x) = 12x - 3x^2$

$0 = 3x(4-x)$

$x = 0, 4$

	$3x$	$4-x$	f'	f
$(-\infty, 0)$	$-$	$+$	$-$	Dec
$(0, 4)$	$+$	$+$	$+$	Inc
$(4, \infty)$	$+$	$-$	$-$	Dec

LOCAL MAX.

$(4, 32)$

LOCAL Min.

$(0, 0)$

$(4,32)$

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