Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x)=[g(x)]^n$$

$g(x)=(x^3+3)^2$

$g'(x)=2(x^3+3)^1(3x^2)$

$$f'(x)=n[g(x)]^{n-1}g'(x)$$

$f(x)=\frac{1}{3}(3x+7)^4$

$=\frac{1}{3}(4(3x+7)^3(3))$

Examples:

$[g(x)]^n \quad g'(x)$

(1) $F(x)=\frac{1}{8}(8x-5)^9(8)$

$g(x) \quad g'(x)$

$f(x)=\frac{1}{8}\left[\frac{1}{10}(8x-5)^{10}\right]+C$

$f(x)=\frac{1}{80}(8x-5)^{10}+C$

$f'(x)=\frac{1}{8}(8x-5)^9(8)$ ✓

(2) $F(x)=x^3\sqrt{1-5x^4}$

$F(x)=-\frac{1}{20}(1-5x^4)^{1/2}(-20x^3)$

$u^n \cdot du$

$f(x)=-\frac{1}{20}\left[\frac{2}{3}(1-5x^4)^{3/2}\right]+C$

$=-\frac{1}{30}(1-5x^4)^{3/2}+C$

(3) $F(x)=\dfrac{x^3}{\sqrt[3]{1-x^4}}$

$F(x)=-\frac{1}{4}(1-x^4)^{-1/3}(-4x^3)$

$u^n \cdot du$

$f(x)=-\frac{3}{8}(1-x^4)^{2/3}+C$

(4) $F(x)=\dfrac{(5-3x^{-4})^5}{2x^5}$

$F(x)=\frac{1}{24}(5-3x^{-4})^5\, 12x^{-5}$

$u^n \cdot du$

$f(x)=\frac{1}{144}(5-3x^{-4})^6+C$

Antiderivatives Involving Trigonometry

Remember the rules to differentiate trigonometric functions...

$\frac{d}{du}(\sin u) = \cos u \bullet du$	$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$
$\frac{d}{du}(\cos u) = -\sin u \bullet du$	$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$
$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$	$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$

Antiderivatives would just be in the opposite direction

Examples:

Determine the general antiderivative:

(1) $f(x) = \sec^2 x - \cos x + 2\csc x \cot x$

$\sec^2 u \cdot du \qquad -\cos u \cdot du \qquad \csc u \cot u \, du$

$F(x) = \tan x - \sin x - 2\csc x + C$

(2) $f(x) = \cos 5x - x^2 \csc^2 x^3 + 5x \sin 2x^2$

$f(x) = \frac{1}{5}\cos 5x\,(5) - \frac{1}{3}\csc^2 x^3 (3x^2) + \frac{5}{4}\sin 2x^2 (4x)$

$\cos u \cdot du \qquad \csc^2 u \cdot du \qquad \sin u \cdot du$

$F(x) = \frac{1}{5}\sin 5x + \frac{1}{3}\cot x^3 - \frac{5}{4}\cos 2x^2 + C$

3) $f(x) = \sec 7x^3 \tan 7x^3 (5x^2)$

$f(x) = \frac{5}{21}\sec 7x^3 \tan 7x^3 (21x^2)$

$F(x) = \frac{5}{21}\sec 7x^3 + C$

4) $f(x) = \sqrt{1 - \tan x^5} \bullet \sec^2 x^5 \bullet 7x^4$

$f(x) = \frac{7}{-5}(1 - \tan x^5)^{1/2} \sec^2 x^5 (-5x^4)$

$F(x) = -\frac{14}{15}(1 - \tan x^5)^{3/2} + C$

Antiderivatives Involving Inverse Trig Ratios

Remember the rules to differentiate inverse trigonometric functions...

$$\frac{d\left(\sin^{-1}u\right)}{du}=\frac{1}{\sqrt{1-u^2}}du \qquad \frac{d\left(\csc^{-1}u\right)}{du}=\frac{-1}{u\sqrt{u^2-1}}du$$

$$\frac{d\left(\cos^{-1}u\right)}{du}=\frac{-1}{\sqrt{1-u^2}}du \qquad \frac{d\left(\sec^{-1}u\right)}{du}=\frac{1}{u\sqrt{u^2-1}}du$$

$$\frac{d\left(\tan^{-1}u\right)}{du}=\frac{1}{1+u^2}du \qquad \frac{d\left(\cot^{-1}u\right)}{du}=\frac{-1}{u^2+1}du$$

Examples:

Determine the general antiderivative:

$(5x^6+1)^{-1}$

(1) $F(x)=\dfrac{5}{\sqrt{1-9x^2}}$

$F(x)=\dfrac{5}{3}\left(\dfrac{(3)}{\sqrt{1-(3x)^2}}\right)$

$f(x)=\dfrac{5}{3}\sin^{-1}(3x)+C$

$\dfrac{x^2}{5x^6+1}$

$(\sqrt{5}x^3)^2$

(2) $F(x)=\dfrac{-5x^2}{5x^6+1}$

$F(x)=-\dfrac{5}{3\sqrt{5}}\dfrac{(3\sqrt{5})x^2}{(\sqrt{5}x^3)^2+1}$

$f(x)=-\dfrac{5}{3\sqrt{5}}\tan^{-1}(\sqrt{5}x^3)+C$

$\dfrac{5}{3\sqrt{5}}\left(\dfrac{\sqrt{5}}{\sqrt{5}}\right)$

$\dfrac{5\sqrt{5}}{15}=\dfrac{\sqrt{5}}{3}$

or

$=\dfrac{5}{3\sqrt{5}}\cot^{-1}(\sqrt{5}x^3)+C$

Antiderivatives involving logarithms and exponential functions...

Remember the following derivative rules...

$$f(x) = \ln[g(x)] \qquad f(x) = e^{g(x)}$$

$$f'(x) = \frac{1}{g(x)} g'(x) \qquad f'(x) = e^{g(x)} \bullet g'(x)$$

$d(\ln u) = \frac{du}{u}$

Examples:

$\frac{2}{7}\left(\frac{1}{x}\right)$

(1) $f'(x) = \dfrac{2}{7x}$ $= \frac{2}{7}\ln|x| + C$

$\frac{du}{u} = \left(\frac{7}{7x}\right)\frac{(2)}{7}$ $\frac{2}{7}\left(\frac{1}{x}\right)$

$f(x) = \frac{2}{7}\ln|7x| + C \Rightarrow \frac{2}{7}\left(\frac{7}{7x}\right)$

(2) $f'(x) = \dfrac{3x^2}{x^3+1}$ $\left(\frac{1}{3}\right)$

$f(x) = \frac{1}{3}\ln|x^3+1| + C$

$f(x) = \ln\sqrt[3]{x^3+1} + C$

(3) $f'(x) = \dfrac{-5\sec^2 3x}{(3)\tan 3x}$ (3)

$f(x) = -\frac{5}{3}\ln|\tan 3x| + C$

$\frac{1}{7}\ln|\sec 7x|$

$\frac{1}{7}$ $\frac{1}{\sec 7x}\sec 7x\tan 7x\ (7)$

(4) $f'(x) = \tan 7x$

$f'(x) = \frac{\sin 7x}{-7\cos 7x}(-7)$

$f(x) = -\frac{1}{7}\ln\cos|7x| + C$

$= \frac{1}{7}\ln[\cos|7x|]^{-1} + C$

$= \frac{1}{7}\ln\left(\frac{1}{\cos 7x}\right) + C$

$= \frac{1}{7}\ln\sec|7x| + C$

(5) $f'(x) = 3xe^{5x^2}$ $d(e^u) = e^u \cdot du$

$f'(x) = \frac{3}{10} e^{5x^2}(10x)$

$e^u \cdot du$

$f(x) = \frac{3}{10}e^{5x^2} + C$

(6) $f'(x) = 7x^2 e^{x^3}$

$f'(x) = \frac{7}{3} e^{x^3}(3x^2)$

$= \frac{7}{3}e^{x^3} + C$

(7) $f'(x) = \dfrac{e^{\sqrt{x}}}{\sqrt{x}}$

$f'(x) = 2e^{x^{1/2}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$f(x) = 2e^{\sqrt{x}} + C$

(8) $f'(x) = \frac{-1}{5}\csc 5x \cot 5x \left(e^{\csc 5x}\right)(-5)$

$f(x) = -\frac{1}{5}e^{\csc 5x} + C$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

1. $f'(x) = 2x - \cos x + 1, \quad f(0) = 3$

$f(x) = x^2 - \sin x + x + C$

$3 = 0 - \sin 0 + 0 + C$

$C = 3$

$f(x) = x^2 - \sin x + x + 3$

2. $f''(x) = 12x^2 + 6x - 4, \quad f(0) = 4 \; and \; f(1) = 1$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

$f(x) = x^4 + x^3 - 2x^2 + Cx + K$

$4 = K$

$1 = 1 + 1 - 2 + C + 4$

$C = -3$