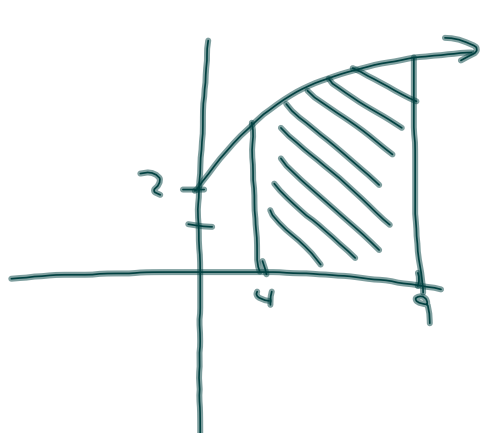# Warm Up

Use a definite integral to evaluate the following areas:

1. Determine the area below the curve $f(x) = \sqrt{x} + 2$ and above the x-axis between $x = 4$ and $x = 9$.
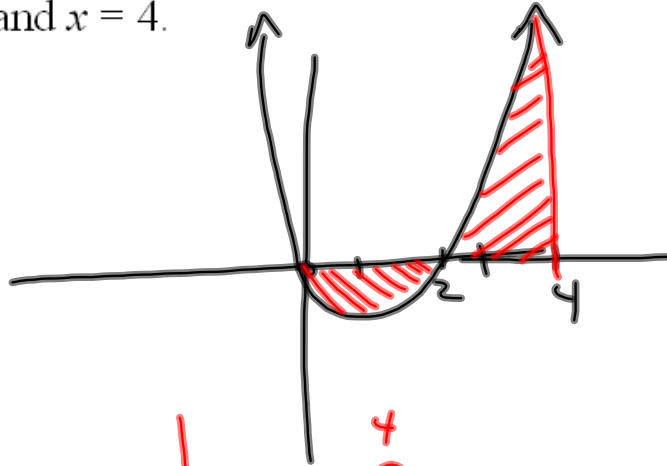
$$\int_4^9 (\sqrt{x} + 2)\, dx$$

$$= \frac{2}{3} x^{3/2} + 2x \Big|_4^9$$

$$= \left[\frac{2}{3}(9)^{3/2} + 2(9) \right] - \left[\frac{2}{3}(4)^{3/2} + 2(4) \right]$$

$$= (18 + 18) - \left(\frac{16}{3} + 8 \right)$$

$$= 36 - 8 - \frac{16}{3}$$

$$= \frac{28}{1} - \frac{16}{3} = \boxed{\frac{68}{3} \, u^2}$$

2. Determine the area bounded by the curve $f(x)=x^2-2x$ and the x-axis between $x=0$ and $x=4$.

$$y=x^2-2x^2$$
$$0=x(x-2)$$
$$x=0,2$$

$$A = \left| \int_0^2 (x^2-2x)\,dx \right| + \int_2^4 (x^2-2x)\,dx$$

$$\left(\frac{x^3}{3} - x^2 \right)\Big|_0^2 + \left(\frac{x^3}{3} - x^2 \right)\Big|_2^4$$

$$\left(\frac{8}{3} - 4 \right) \qquad \left[\left(\frac{64}{3} - 16 \right) - \left(\frac{8}{3} - 4 \right) \right]$$
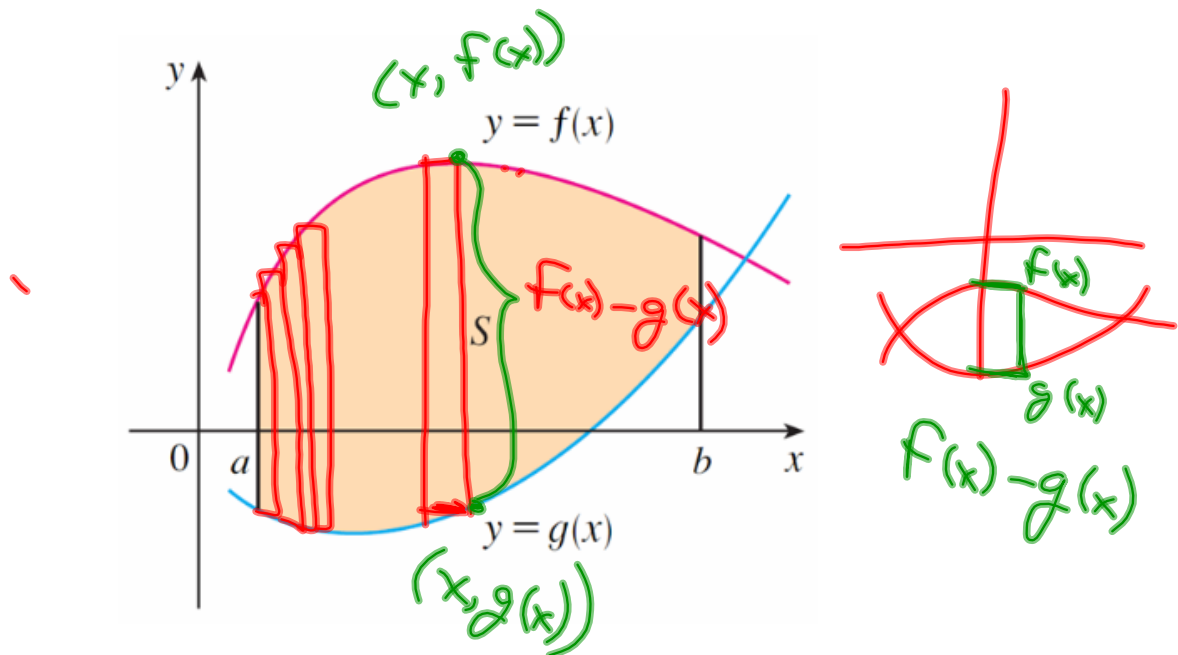
$$\left| \frac{-4}{3} \right| \qquad + \quad \frac{56}{3} - 12$$
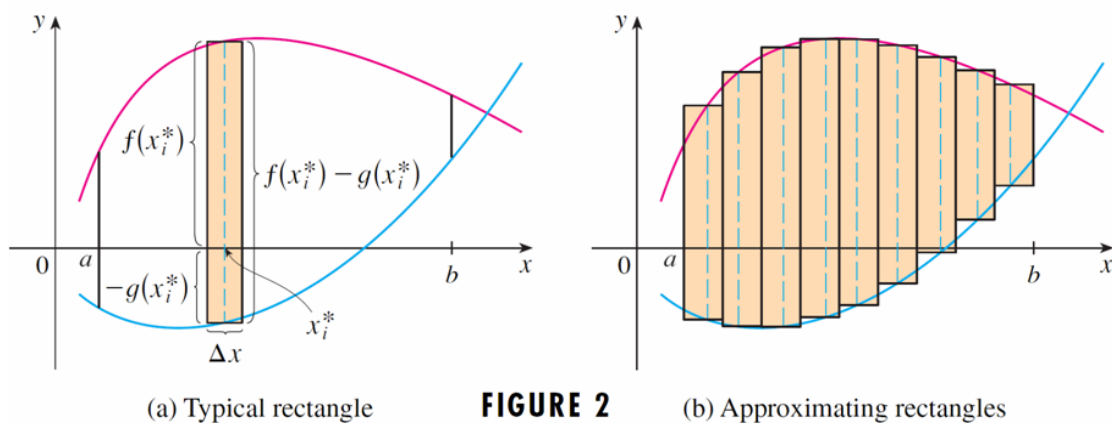
$$= \frac{60}{3} - 12$$

$$= 20 - 12$$

$$= 8u^2$$

Area Between Curves

What if we would like to determine the area between two curves?

The figure shows region S bounded above by $y = f(x)$ and below by $y = g(x)$, with point $(x, f(x))$ on the upper curve and $(x, g(x))$ on the lower curve, and the vertical distance $f(x) - g(x)$ between them, over the interval from a to b.

Could we use rectangular strips?

(a) Typical rectangle: shows $f(x_i^*)$, $-g(x_i^*)$, $f(x_i^*) - g(x_i^*)$, x_i^*, and Δx.

FIGURE 2

(b) Approximating rectangles

Write out a definite integral that would represent the area of region S.

Example:

Determine the area bounded by the curve $f(x) = -x^2 - 2$ and the lines $x = -1$, $x = 2$ and $y = -4$.
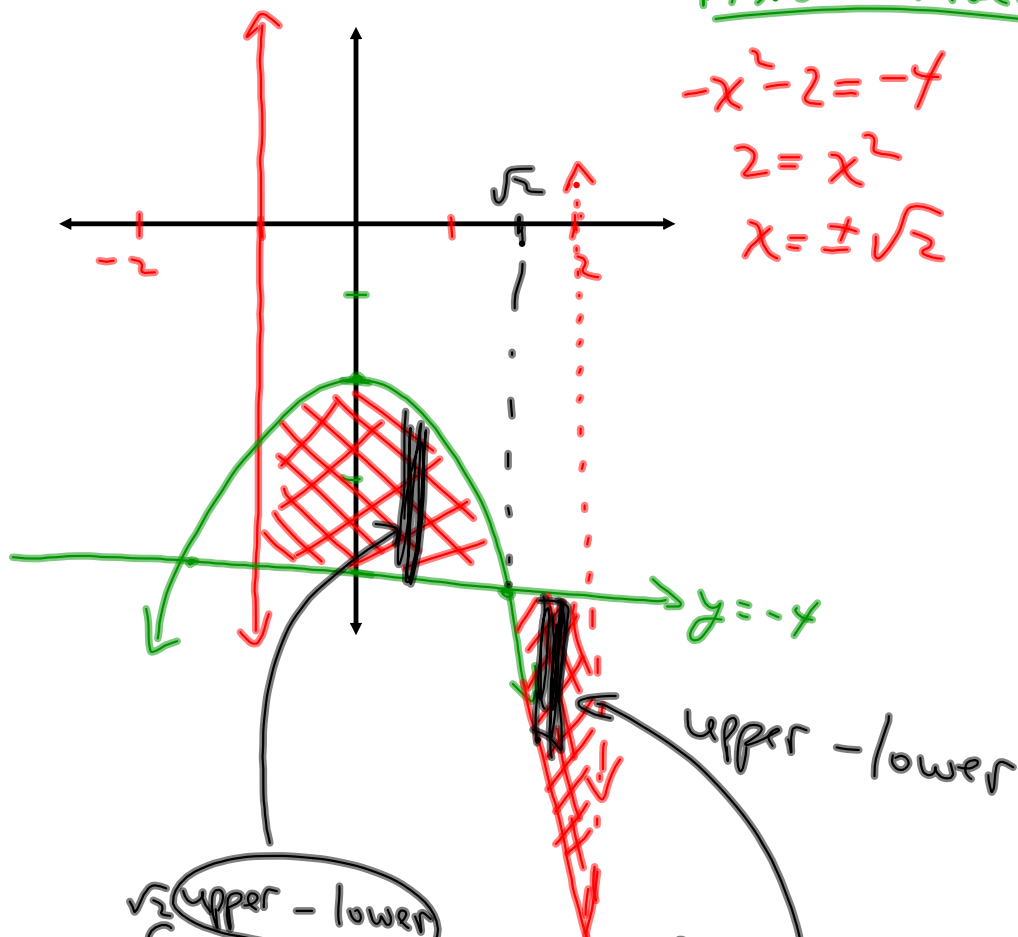
1. Start with a sketch...

Pts. of Intersection

$$-x^2 - 2 = -4$$
$$2 = x^2$$
$$x = \pm\sqrt{2}$$

$y = -4$

upper − lower

upper − lower

$$\int_{-1}^{\sqrt{2}} \left[(-x^2-2)-(-4)\right]dx + \int_{\sqrt{2}}^{2}\left[-4-(-x^2-2)\right]dx$$

$$\int_{-1}^{\sqrt{2}}(-x^2+2)dx + \int_{\sqrt{2}}^{2}(x^2-2)dx$$

$$= \left(-\frac{x^3}{3}+2x\right)\Big|_{-1}^{\sqrt{2}} + \left(\frac{x^3}{3}-2x\right)\Big|_{\sqrt{2}}^{2}$$

$=$

4

Example:

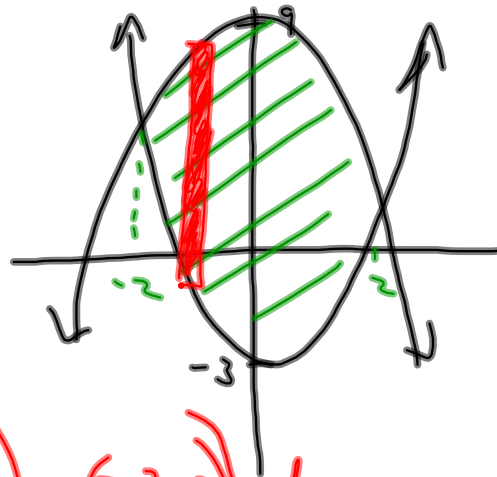Determine the area bounded by the curves $f(x) = -x^2 + 9$ and $f(x) = 2x^2 - 3$.

Pts. of Intersection:

$$-x^2 + 9 = 2x^2 - 3$$

$$-3x^2 = -12$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

$$\int_{-2}^{2} \left((-x^2 + 9) - (2x^2 - 3) \right) dx$$

$$\int_{-2}^{2} (-3x^2 + 12) dx$$

$$= -x^3 + 12x \Big|_{-2}^{2}$$

$$= (-8 + 24) - (8 - 24)$$

$$= \underline{32u^2}$$