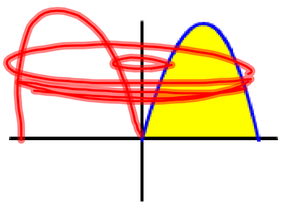
Another means of calculating volume...

Cylindrical Shell Method:

Consider the area bounded by the graph of the function $f(x) = x - x^2$ and the x-axis.

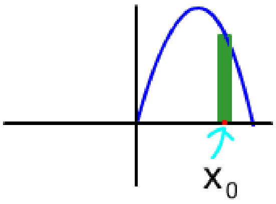


$$y = x - x^2 \Rightarrow y = -\left(x^2 - x + \tfrac{1}{4}\right) + \tfrac{1}{4}$$

$-\tfrac{1}{4}$

$$y = -\left(x - \tfrac{1}{2}\right)^2 + \tfrac{1}{4}$$

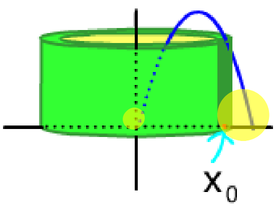
$$\sqrt{-y + \tfrac{1}{4}} = \sqrt{\left(x - \tfrac{1}{2}\right)^2}$$



Choose some x_0 between 0 and 1. Draw a rectangle with height $f(x_0)$ and with very small width Δx.

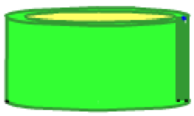
$$\pm\sqrt{\tfrac{1}{4} - y} = x - \tfrac{1}{2}$$

$$x = \tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} - y}$$

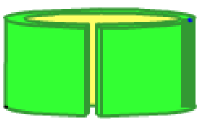


Rotate the rectangle about the y-axis.

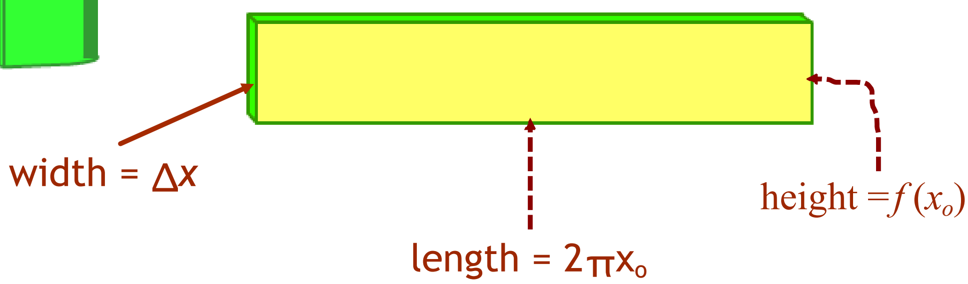
The result is a cylinder with a "very small side" like the side of a can:

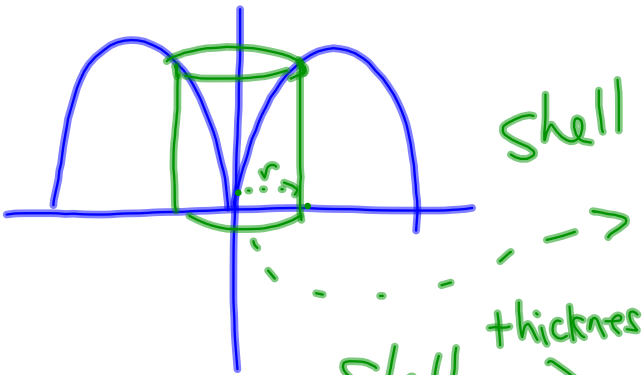


Take this cylinder and cut it vertically as shown:

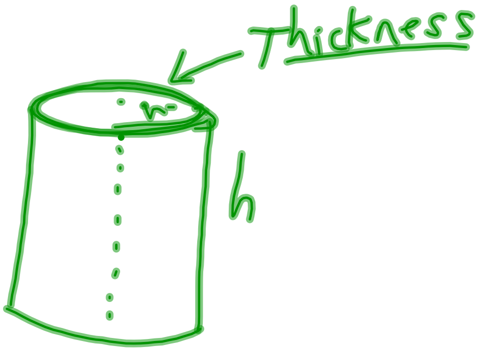


and stretch it out "flat":

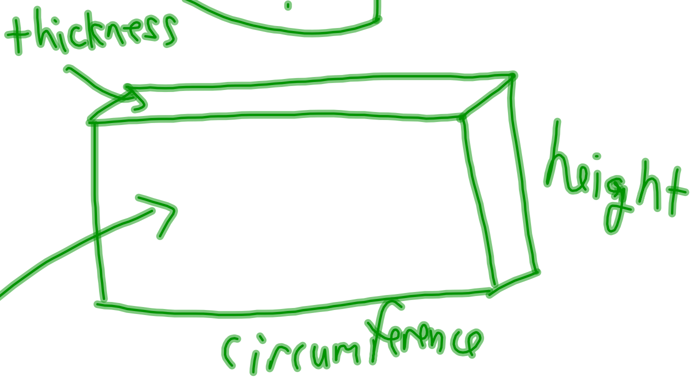




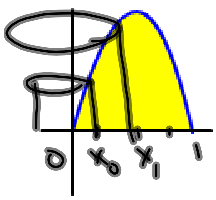
Shell



Shell
(laying flat)



Volume of a Shell $= 2\pi r h \,(dy \text{ or } dx)$



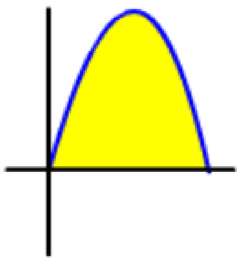
The volume of the solid obtained by rotating this area about the y-axis is:

$$2\pi \int_0^1 x\, f(x)\, dx = 2\pi \int_0^1 x\,(x - x^2)\, dx$$

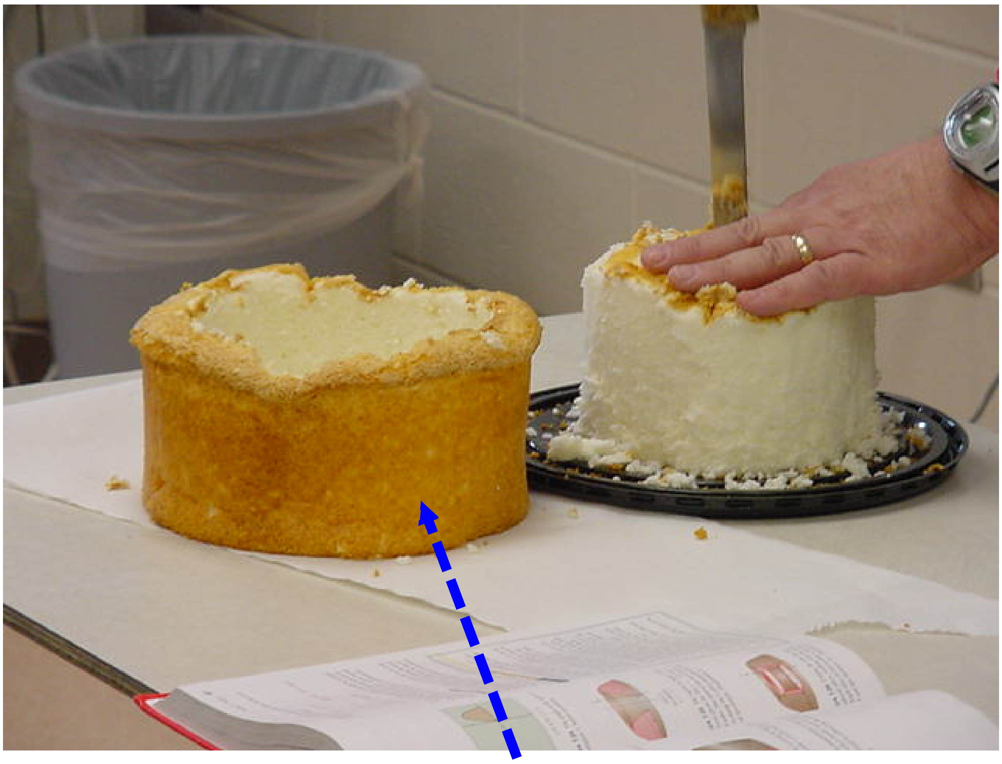
radius height thickness

Notice that even though we are rotating about a vertical line, the integral is still in terms of *x*.

What if we had used cylindrical disks?



Imagine cutting a cake using this method...



This would be an example of a cylindrical shell

Example 1:

The region bounded by the curve $y = 4 - x^2$, $y = x$, and $x = 0$ is revolved about the y-axis to generate a solid. Determine the volume of this solid.

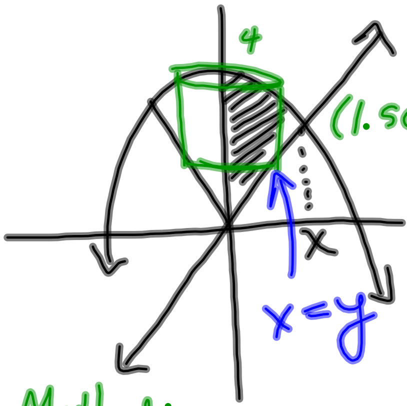
$4 - x^2 = x$

$x^2 + x - 4 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(1)(-4)}}{2}$

$x = \frac{-1 \pm \sqrt{17}}{2}$

$\underline{x = 1.562}$



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \pm\sqrt{4 - y}$

Shell Method:

$2\pi \int_0^{1.562} x\left[4 - x^2 - x\right] dx$

$2\pi \int_0^{1.562} \left(4x - x^3 - x^2\right) dx$

$= 2\pi \left[2x^2 - \frac{x^4}{4} - \frac{x^3}{3}\right]\Big|_0^{1.562}$

$= \underline{13.327 \, u^3}$

Washers...

$\pi \int_0^{1.562} y^2 \, dy + \pi \int_{1.562}^{4} \left(\sqrt{4 - y}\right)^2 dy$

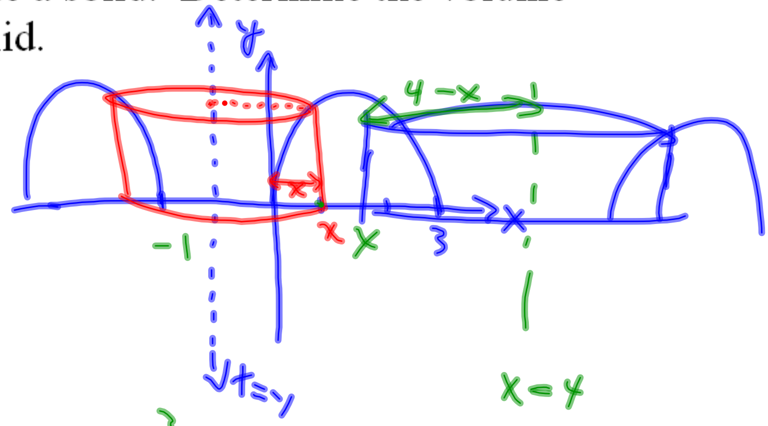
Example 2:

The region bounded by the curve $y = 3x - x^2$ and the x-axis is revolved about the line x = -1 to generate a solid. Determine the volume of this solid.

$y = 3x - x^2$

$y = x(3-x)$

$x = 0, 3$



$$V = 2\pi \int_0^3 (x+1)(3x-x^2)\,dx$$

$$V = 2\pi \int_0^3 (3x^2 - x^3 + 3x - x^2)\,dx$$

$$= 2\pi \int_0^3 (2x^2 - x^3 + 3x)\,dx$$

$$= 2\pi \left(\frac{2}{3}x^3 - \frac{x^4}{4} + \frac{3}{2}x^2\right)\Big|_0^3$$

$$= 2\pi \left(\frac{2}{3}(27) - \frac{81}{4} + \frac{3}{2}(9)\right)$$

$$= 2\pi \left(\frac{18}{1} - \frac{81}{4} + \frac{27}{2}\right)$$

$$= 2\pi \left(\frac{72}{4} - \frac{81}{4} + \frac{54}{4}\right)$$

$$= \frac{90\pi}{4}$$

$$= \frac{45\pi}{2} u^3$$

Practice Questions...

Worksheet: Volume using shell method

Textbook:
Pg. 457
#1-16

Attachments

volume using shell method worksheet.doc