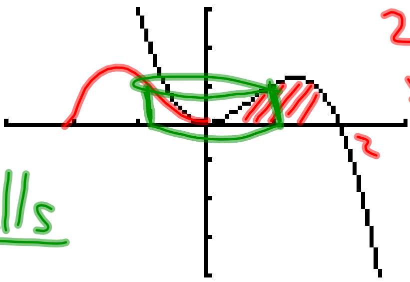
Warm Up

Determine the volume of the solid obtained by rotating the region enclosed by the curve $f(x) = 2x^2 - x^3$ and the x-axis about the y-axis.

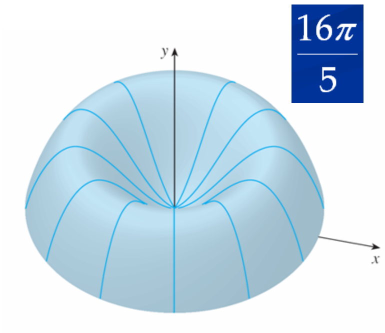
$\frac{16\pi}{5}$



$2x^2 - x^3 = 0$

$x^2(2-x) = 0$

$x = 0, 2$



Shells

$$V = 2\pi \int_0^2 x(2x^2 - x^3)\,dx$$

$$V = 2\pi \int_0^2 (2x^3 - x^4)\,dx \Rightarrow 2\pi\left(\tfrac{1}{2}x^4 - \tfrac{x^5}{5}\right)\Big|_0^2$$

$$= 2\pi\left(8 - \tfrac{32}{5}\right) - 0$$

$$= 2\pi\left(\tfrac{8}{5}\right)$$

$$= \frac{16\pi}{5}$$

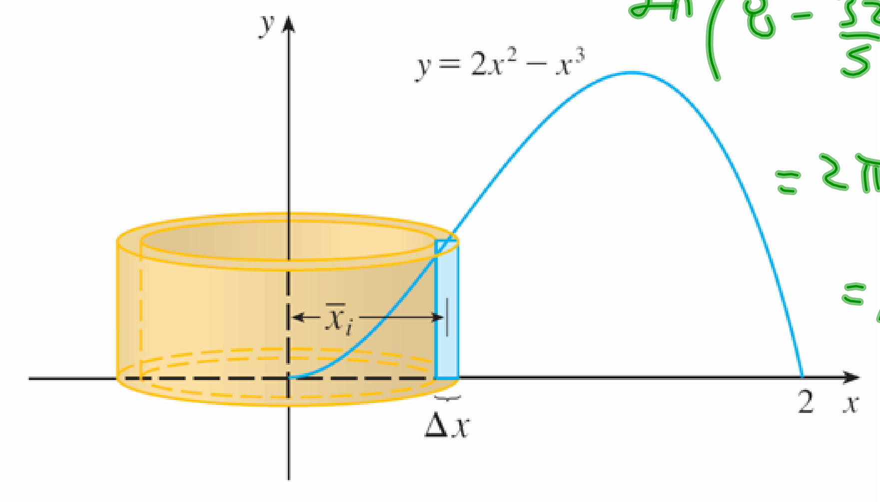
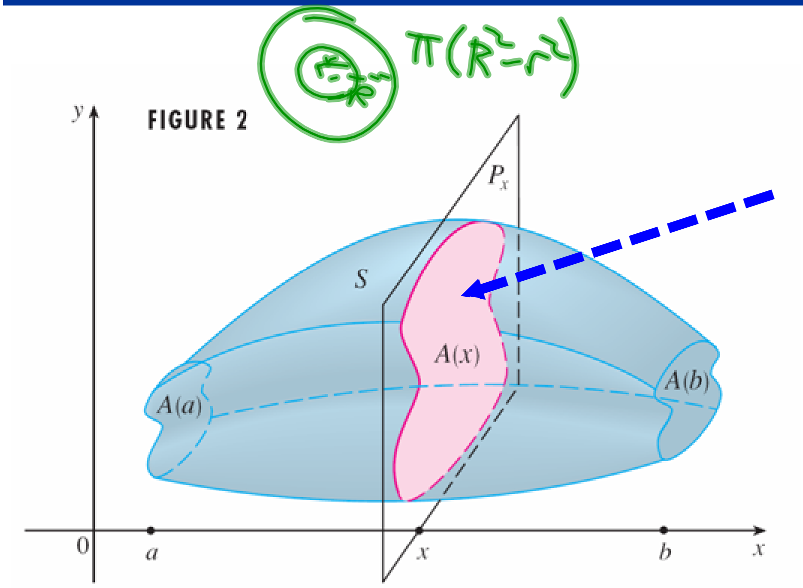


FIGURE 18 A cylindrical shell

Volume: Using known cross-sectional areas

- The formula $V=\int_a^b A(x)\,dx$ can be applied to *any* solid for which the cross-sectional area $A(x)$ can be found

$\pi(R^2-r^2)$



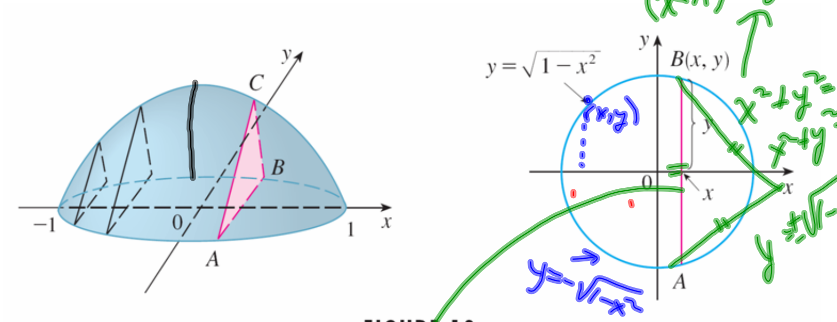
Notice that each cross-section would be the same shape. If we could determine the area of each of these cross-sections, we could determine the volume of this solid.

Think of it as summing the volume of each slice of bread to determine the volume of the loaf of bread!!

Example:

A solid has a circular base of radius 1 . Parallel cross-sections perpendicular to the base are equilateral triangles. Find the volume of the solid.

$(x-h)^2+(y-k)^2=r^2$



(a) The solid FIGURE 13 (b) Its base

$x^2+y^2=r^2$

$x^2+y^2=1$

$y=\pm\sqrt{1-x^2}$

(x,y)

$y=-\sqrt{1-x^2}$

Cross-Section

$60°$

$2\sqrt{1-x^2}$

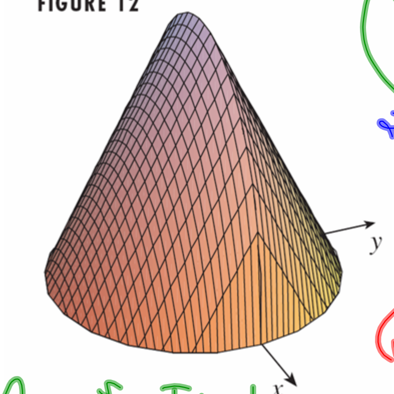
h

$A=\frac{1}{2}\left(2\sqrt{1-x^2}\right)^2\sin 60°$

$A=\frac{1}{2}4(1-x^2)\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{A=\sqrt{3}(1-x^2)}$

FIGURE 12



$(\sqrt{1-x^2})^2+h^2=(2\sqrt{1-x^2})^2$

$h^2=4(1-x^2)-(1-x^2)$

$h^2=3(1-x^2)$

$h=\sqrt{3}(\sqrt{1-x^2})$

Area of a Triangle

$A=\frac{1}{2}bh$ or $A=\frac{1}{2}ab\sin\theta$

a

θ

b

$A=\frac{1}{\cancel{2}}\cancel{2}(\sqrt{1-x^2})\sqrt{3}(\sqrt{1-x^2})$

$\boxed{A=\sqrt{3}(1-x^2)}$

$V=\sqrt{3}\int_{-1}^{1}(1-x^2)\,dx$ or $2\sqrt{3}\int_{0}^{1}(1-x^2)\,dx$

$V=\sqrt{3}\left(x-\frac{x^3}{3}\right)\Big|_{-1}^{1}$

$V=\sqrt{3}\left[\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right]$

$V=\sqrt{3}\left(2-\frac{2}{3}\right)$

$V=\frac{4\sqrt{3}}{3}u^3$

5.1.4 [Activity] A boneless baked turkey breast that is ten inches long from one end to the other is sliced up in to very thin slices. Each slice has a cross-sectional area of $(-x^2 + 10x)$ square inches for each x between *0* and *10*. What is the volume of the turkey breast?

$$V = \int_0^{10} (-x^2 + 10x)\,dx$$

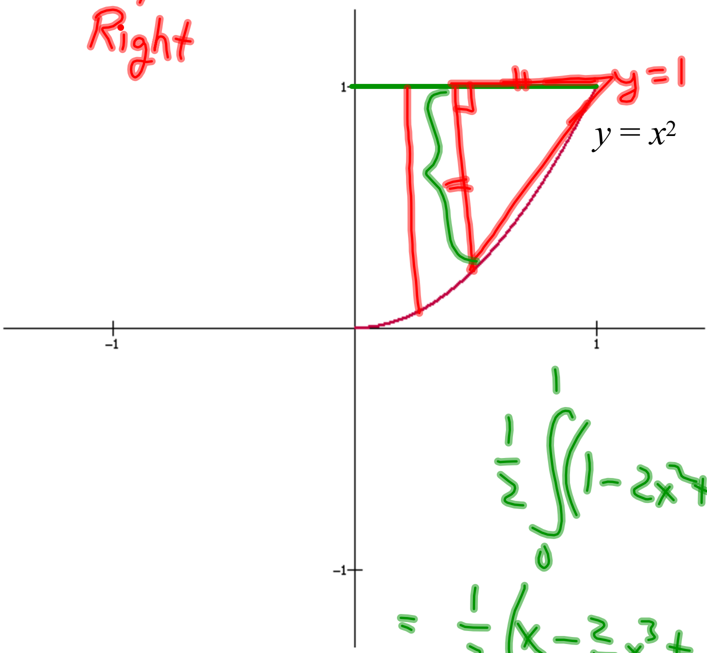
$$= -\frac{x^3}{3} + 5x^2 \Big|_0^{10}$$

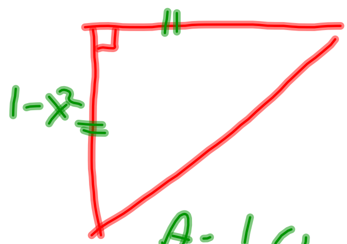
$$= \left(-\frac{1000}{3} + 500\right) - 0$$

$$= \frac{500}{3} \text{ in}^3$$

Determine the volume of the solid S, whose base is the parabolic region $f(x) = x^2$ over the closed interval $0 \leq x \leq 1$, and whose cross-sections perpendicular to the x-axis are isosceles triangles with height equal to the base. solution: 4/15 u^3

Right





$A = \frac{1}{2}(1-x^2)^2$

$$\frac{1}{2}\int_0^1 (1-2x^2+x^4)\,dx$$

$$= \frac{1}{2}\left(x - \frac{2}{3}x^3 + \frac{x^5}{5}\right)\Big|_0^1$$

$$= \frac{1}{2}\left(1 - \frac{2}{3} + \frac{1}{5}\right)$$

$$= \frac{1}{2}\left(\frac{15-10+3}{15}\right) = \frac{1}{2}\left(\frac{8}{15}\right) = \frac{4}{15}u^3$$

The base of the volume is the region bounded by the curves $y = 8 - x^2$ and $y = x^2$.
The cross sections perpendicular to the x-axis are:

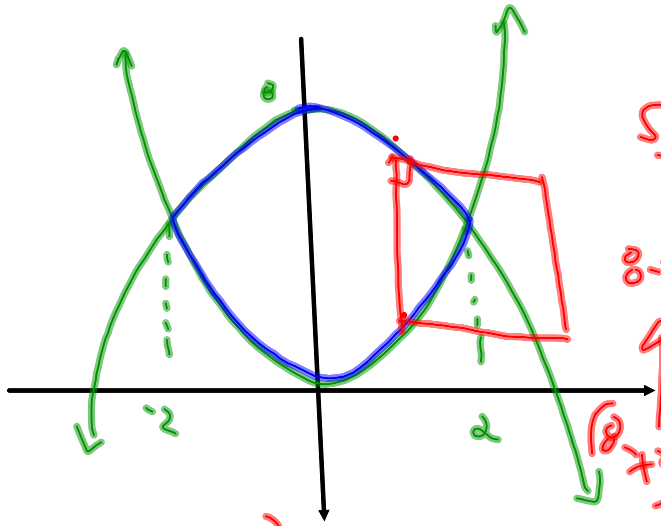
a. Squares
b. Equilateral triangles
c. Isosceles right triangles with leg on the base
d. Isosceles right triangles with hypotenuse on the base
e. Semi-circles
f. Quarter-circles
g. Isosceles trapezoids with 60° base angles

$8 - x^2 = x^2$

$8 = 2x^2$

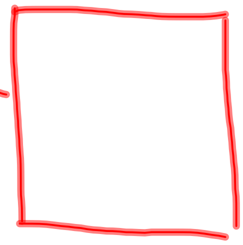
$4 = x^2$

$\pm 2 = x$



Squares

$8-2x^2$



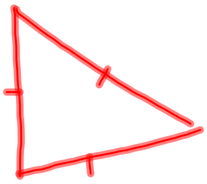
$(8-x^2)-x^2$

$A = (8-2x^2)^2$

$$V = 2\int_0^2 (64 - 32x^2 + 4x^4)\,dx$$

b)

$8-2x^2$



$A = \frac{1}{2}(8-2x^2)^2 \sin 60^\circ$

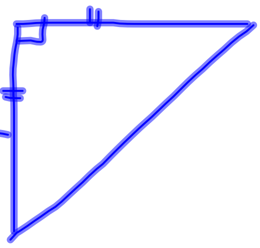
$A = \frac{1}{2}(8-2x^2)^2 \frac{\sqrt{3}}{2}$

$A = \frac{\sqrt{3}}{4}(8-2x^2)^2$

$$V = \frac{\sqrt{3}}{4}\int_{-2}^{2} (8-2x^2)^2\,dx$$

c)

$8-2x^2$



$A = \frac{1}{2}(8-2x^2)^2$

$$V = \frac{1}{2}\int_{-2}^{2} (8-2x^2)^2\,dx$$

d)