Warm UP

The brakes of a car decelerate the car at 22 ft./s^2 The car is travelling at 60 mph and applies the brakes 175 feet from a concrete barrier. Should we call 911? 1 mile = 5280 feet

$$\frac{60\text{ miles}}{\text{hr}} \cdot \frac{5280\text{ ft.}}{1\text{ mile}} \times \frac{1\text{ hr}}{3600\text{ sec}} = 88\ \text{ft/sec}$$

$a = -22$

$v = -22t + C$

$88 = -22(0) + C$

$v = -22t + 88$

$t = 0$
$v = 88$
$d = 0$

$d = -11t^2 + 88t + C$

$C = 0$

$d = -11t^2 + 88t$

time to stop... $-22t + 88 = 0$

$-22t = -88$

$t = 4$ sec

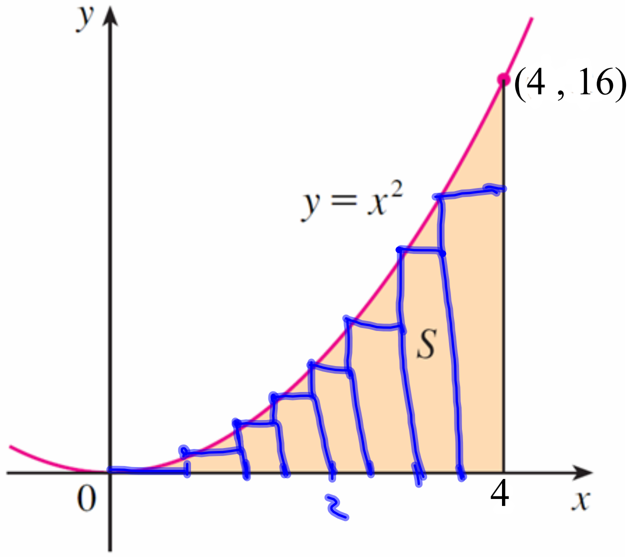
How far was skid??

$d = -11(4)^2 + 88(4)$

$d = 176$ feet ← Will hit barrier...

BARELY!!

Areas bound by curves...



x	$f(x)$
0	0
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25
4	16

Rectangles

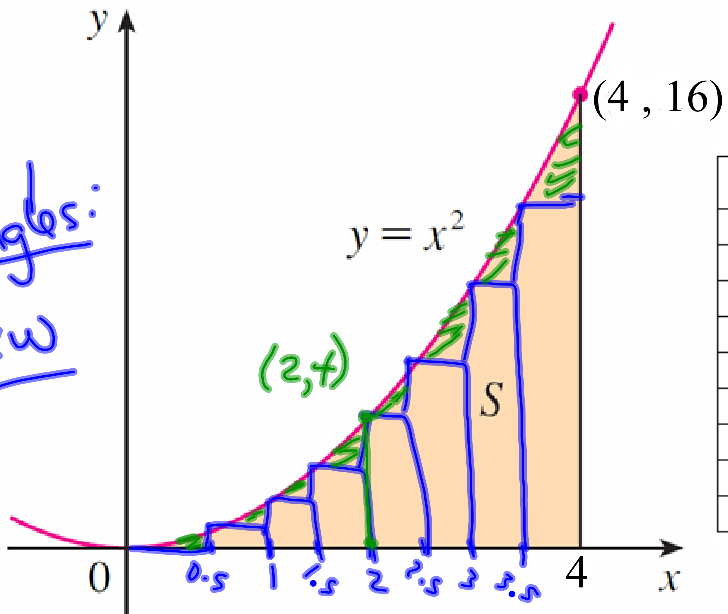
(1) Determine the area of region S by using the **left** endpoints of 8 equal subintervals.

(2) Determine the area of region S by using the **right** endpoints of 8 equal subintervals.

(3) Determine the area of region S by using 8 subintervals and the **trapezoidal rule**.

Rectangles:

$A = l \times w$

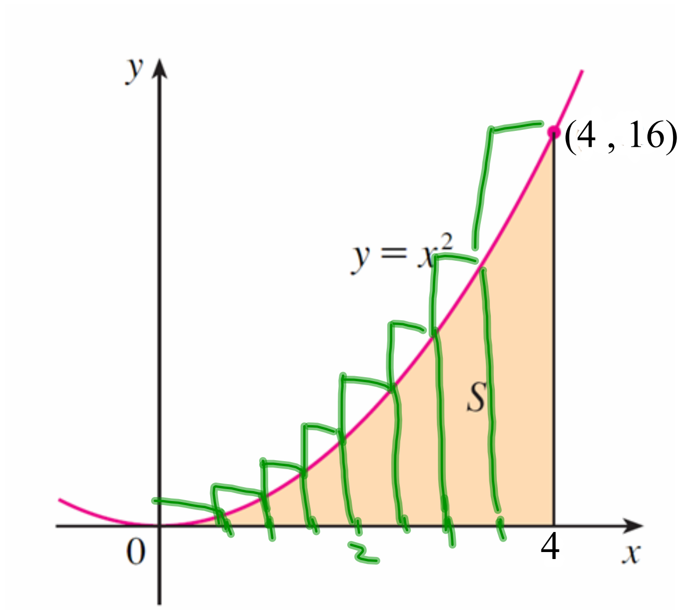


x	$f(x)$
0	0
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25
4	16

(1) Determine the area of region S by using the **left** endpoints of 8 equal subintervals.

$A = 0.5(0 + 0.25 + 1 + 2.25 + 4 + 6.25 + 9 + 12.25)$

$A = 17.5\ u^2$ — underestimate



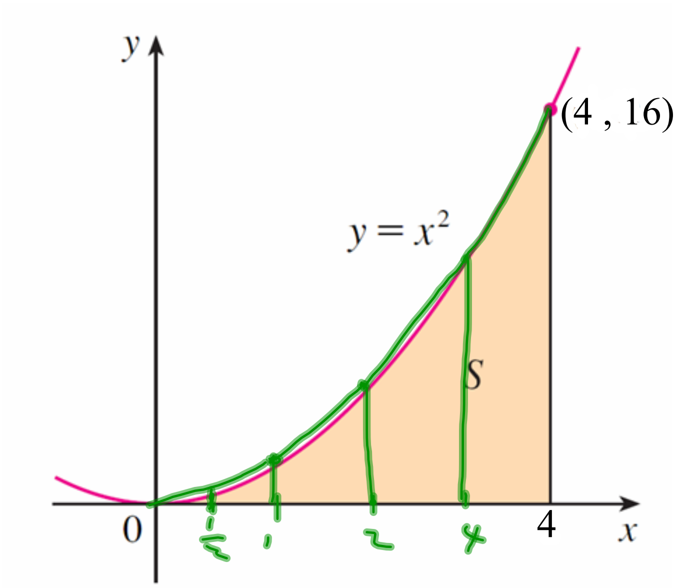
x	$f(x)$
0	0
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25
4	16

(2) Determine the area of region S by using the **right** endpoints of 8 equal subintervals.

$A = 0.5(0.25 + 1 + 2.25 + 4 + 6.25 + 9 + 12.25 + 16)$

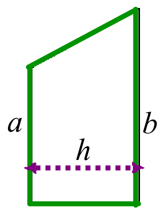
$A = 25.5u^2$

Average: $\dfrac{25.5 + 17.5}{2} = 21.5u^2$



x	$f(x)$
0	0
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25
4	16

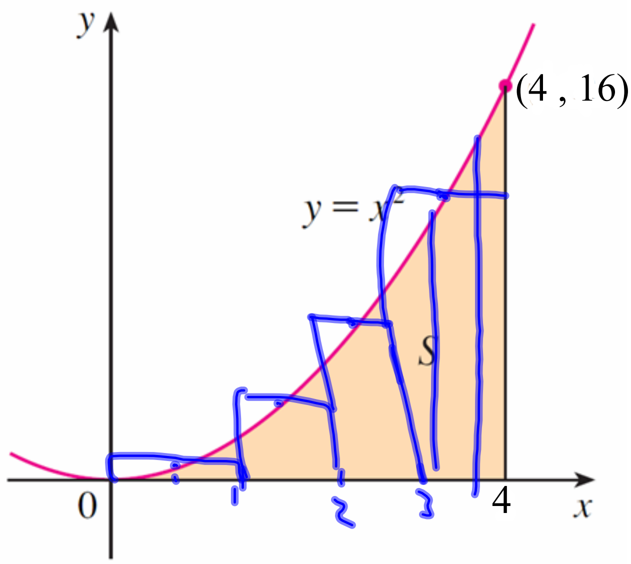
(3) Determine the area of region S by using 8 subintervals and the **trapezoidal rule**.



$$A = \frac{1}{2}(a+b)h$$

$$A = \frac{1}{2}(0.5)\big[(0+0.25)+(0.25+1)+(1+2.25)+(2.25+4)+(4+6.25)+(6.25+9)+(9+12.25)+(12.25+16)\big]$$

$A = 21.5\ u^2$


y
(4 , 16)
y = x²
S
0
1
2
3
4
x

"Sigma"

Sigma Notation

$$\sum_{k=1}^{n} f(x_k)$$

Summation

Example:

Evaluate $\displaystyle\sum_{k=1}^{4}(3k-2)$

$= \overset{k=1}{[3(1)-2]} + \overset{k=2}{[3(2)-2]} + \overset{k=3}{[3(3)-2]} + \overset{k=4}{[3(4)-2]}$

$= 1 + 4 + 7 + 10$

$= 22$

What about the following?

$\displaystyle\sum_{k=1}^{7}(5w)$

$= 5(1)+5(2)+5(3)+5(4)+5(5)+5(6)+5(7)$

$= 140$

$\displaystyle\sum_{k=1}^{5}(-3)$

$= -3 + -3 + -3 + -3 + -3$

$= -15$

Summation properties...

$$\sum_{i=1}^{n} C = Cn \quad , C \in \mathbb{R}$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} Ca_i = C\sum_{i=1}^{n} a_i$$

Evaluate the following...
(i) without summation properties
(ii) using summation properties

$$\sum_{i=1}^{3} \left(2i^2 - 5i\right)$$

$$(1)\ [2(1)^2 - 5(1)] + [2(2)^2 - 5(2)] + [2(3)^2 - 5(3)]$$

$$-3 + -2 + 3$$

$$= -2$$

$$2)\ \sum_{i=1}^{3} 2i^2 - \sum_{i=1}^{3} 5i$$

$$2\sum_{i=1}^{3} i^2 - 5\sum_{i=1}^{3} i$$

$$= 2(1^2 + 2^2 + 3^2) - 5(1+2+3)$$

$$= 2(14) - 5(6)$$

$$= -2$$

Summation Rules

...

$$\sum_{k=1}^{n} k = 1+2+3+4+\ldots+(n-1)+n$$

$$\sum_{k=1}^{n} k^2 = 1+4+9+16+\ldots+(n-1)^2+n^2$$

$$\sum_{k=1}^{n} k^3 = 1+8+27+64+\ldots+(n-1)^3+n^3$$

Notice that these are INFINITE

$$\sum_{k=1}^{n} k = \frac{n^2}{2}+\frac{n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4}{4}+\frac{n^3}{2}+\frac{n^2}{4}$$

Notice that these are FINITE

Let's revisit the example used yesterday...

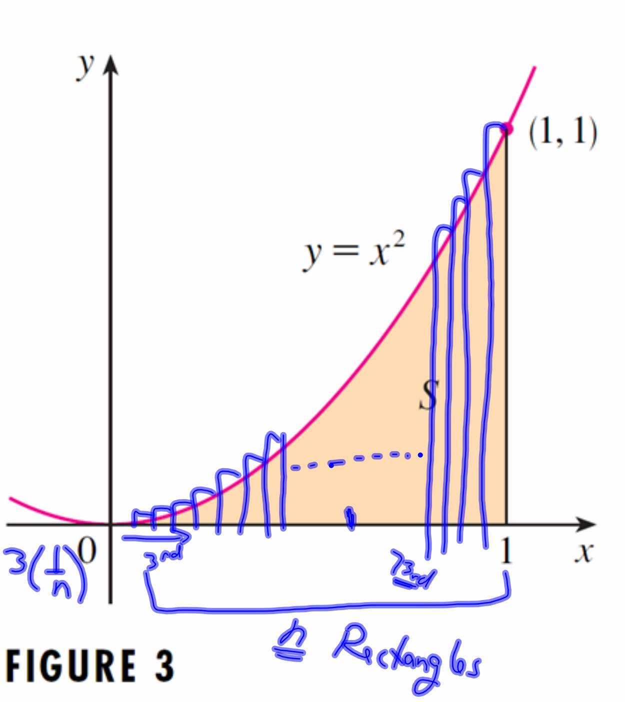


FIGURE 3

We want to find the area below this curve using " n" rectangles.

What will be the width of each rectangle? $\Delta x = \frac{1}{n}$

How will we determine the height of each rectangle? $x_k = k\left(\frac{1}{n}\right)$

$$f(x_k) \longrightarrow f\left(\frac{k}{n}\right)$$

Write out an expression for the area of these "n" rectangles?

$$\sum_{k=1}^{n} \left(\frac{1}{n}\right) f(x_k)$$

$$\sum_{k=1}^{n} \left(\frac{1}{n}\right) f\left(\frac{k}{n}\right)$$

$$= \left(\frac{1}{n}\right) \sum_{k=1}^{n} f\left(\frac{k}{n}\right)$$

$$= \frac{1}{n} \sum_{k=1}^{n} \frac{k^2}{n^2}$$

$$=$$

$$\sum_{k=1}^{n} k = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$f(x) = x^2$

$f\left(\frac{k}{n}\right) = \left(\frac{k}{n}\right)^2$

Attachments

volume using shell method worksheet.doc

Review of antiderivatives, area and volume.doc