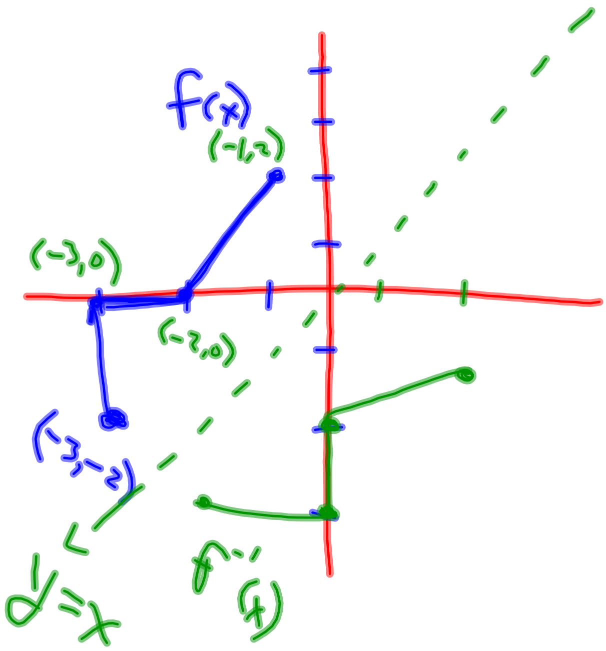


Determine $f^{-1}(x)$ if $f(x) = 3\sqrt{x-5} + 8$

$$x = 3\sqrt{y-5} + 8$$

$$\frac{x-8}{3} = \frac{3\sqrt{y-5}}{3}$$

$$\left(\frac{x-8}{3}\right)^2 = \left(\sqrt{y-5}\right)^2$$

$$\frac{(x-8)^2}{9} = y-5$$

$$y = \frac{1}{9}(x-8)^2 + 5$$

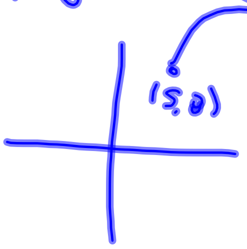
$$f^{-1}(x) = \frac{1}{9}(x-8)^2 + 5$$

Domain & Range of BOTH $f(x)$ & $f^{-1}(x)$

$f(x) = 3\sqrt{x-5} + 8$

D: $x \geq 5$

R: $y \geq 8$



$f^{-1}(x) = \frac{1}{9}(x-8)^2 + 5$

D: $x \geq 8$

R: $y \geq 5$

Practice Problems...

Pages 51 - 55
#2, 3, 5, 6, 8, 9, 11, 15, 18, 20, 21

Combination of Functions

- Two functions f and g can be combined to form new functions
  - $f+g$,
  - $f-g$,
  - fg, and
  - f/g

  just as we add, subtract, multiply, and divide real numbers.

- This is summarized in the following table:

Algebra of Functions Let f and g be functions with domains A and B. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows:

$(f+g)(x) = f(x) + g(x)$	domain $= A \cap B$
$(f-g)(x) = f(x) - g(x)$	domain $= A \cap B$
$(fg)(x) = f(x)g(x)$	domain $= A \cap B$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	domain $= \{x \in A \cap B \mid g(x) \neq 0\}$

"Intersection of"

Overlap

$A \cup B$

"In union with"

Join together

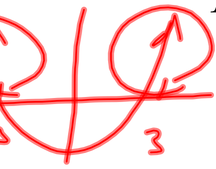
- ***Review of Intersection and Union of two sets:***

$f(x)=\sqrt{x+4}$ $\qquad\qquad$ $g(x)=\sqrt{x^2-9}$

Let A represent the domain of f and B the domain of g.

A: $x+4 \geqq 0$

D: $x \geqq -4$

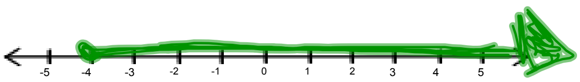


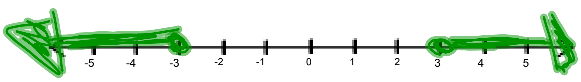
B: $x^2-9 \geqq 0$

$(x-3)(x+3) \geqq 0$

$x=\pm 3$

$x \leqq -3$ or $x \geqq 3$



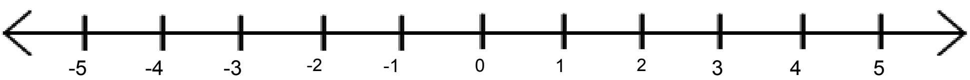


I. Intersection: $A \cap B$

$\{x \mid -4 \leqq x \leqq -3 \ \underline{\text{OR}} \ x \geqq 3, x \in R\}$

II. Union: $A \cup B$

$\underline{x \in R}$



Example

- If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find the functions $f+g$, $f-g$, fg , and f/g .

**Also examine the domain of each of these new functions

$(f+g)(x)=f(x)+g(x)$

$=\sqrt{x}+\sqrt{4-x^2}$

$f(x)=\sqrt{x}$

$D: x \geq 0$

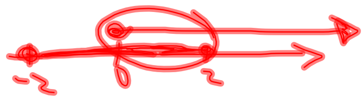
$[0,\infty)\cap[-2,2]$

$g(x)=\sqrt{4-x^2}$

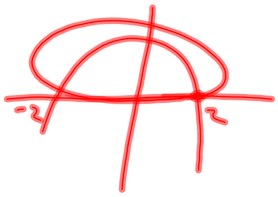
$4-x^2 \geq 0$

$(2-x)(2+x) \geq 0$

$x=\pm 2$



$D: 0 \leq x \leq 2$



$-2 \leq x \leq$

$(f-g)(x)=\sqrt{x}-\sqrt{4-x^2}$

$D: [0,2]$

$(fg)(x)=(\sqrt{x})(\sqrt{4-x^2})$

$=\sqrt{4x-x^3}$

$D: [0,2]$

$(\sqrt{a}\cdot\sqrt{b})=\sqrt{ab}$

$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{\sqrt{4-x^2}}\left(\frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}\right)$

$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

or

$=\sqrt{\frac{x}{4-x^2}}$ $\quad = \frac{\sqrt{4x-x^3}}{4-x^2}$

Can Not include $x=2$

$D: [0,2)$

$\sqrt{a^2+b^2} \neq a+b \leftarrow \sqrt{4-x^2} \neq 2-x$

Compositions of Functions

When the input in a function is another function, the result is called a **composite function.** If

$f(x) = 3x + 2$ and $g(x) = 4x - 5$

then $f[g(x)]$ is a composite function. The statement $f[g(x)]$ is read " f of g of x" or "the composition of f with g." $f[g(x)]$ can also be written as

$$(f \circ g)(x) \quad \text{or} \quad f \circ g(x) \neq (g \circ f)(x)$$

The symbol between f and g is a small open circle. When replacing one function with another, be very careful to get the order correct because compositions of functions are not necessarily commutative (as you'll see).

Example 1

If $f(x) = 3x + 2$ and $g(x) = 4x - 5$, find each of the following.

1. $f[g(4)]$
2. $g \circ f(4)$
3. $f[g(x)]$
4. $(g \circ f)(x)$

1/ $g(4) = 4(4) - 5$
$= 11$
$f(11) = 3(11) + 2$
$= 35$

2/ $f(4) = 3(4) + 2$
$= 14$
$g(14) = 4(14) - 5$
$= 51$

3/ $f(4x-5) = 3(4x-5) + 2$
$= 12x - 13$

4/ $g(3x+2) = 4(3x+2) - 5$
$= 12x + 3$

Example 2

If $f(x) = 3x^2 + 2x + 1$ and $g(x) = 4x - 5$, find each of the following:

1. $f[g(x)]$
2. $g[f(x)]$

1/ $f(4x-5) = 3(4x-5)^2 + 2(4x-5) + 1$

$= 3(16x^2 - 40x + 25) + 8x - 10 + 1$

$= 48x^2 - 120x + 75 + 8x - 9$

$= 48x^2 - 112x + 66$

$g(3x^2+2x+1) = 4(3x^2+2x+1) - 5$

$= 12x^2 + 8x + 4 - 5$

$= 12x^2 + 8x - 1$