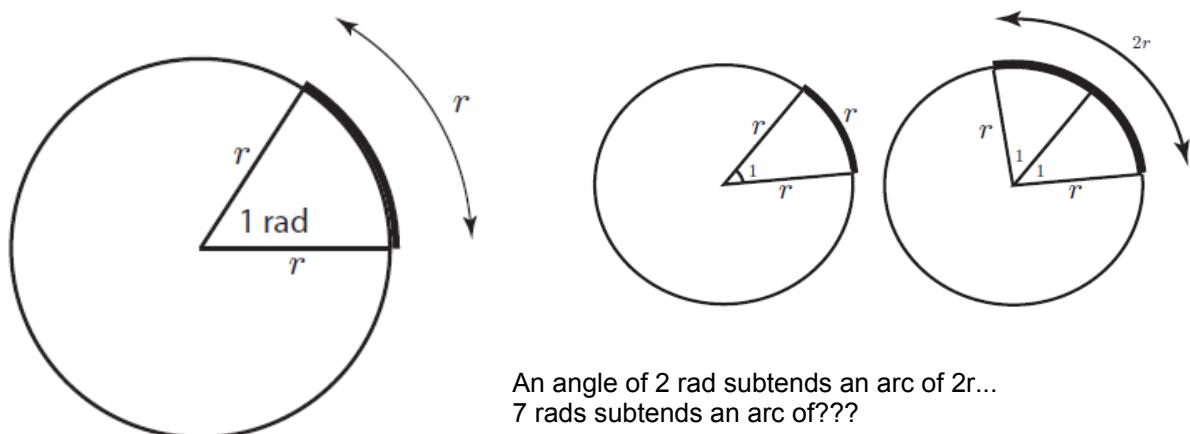
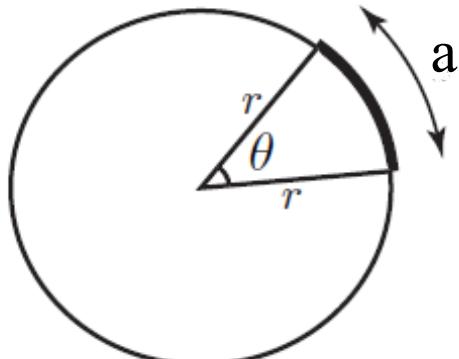


Radian Measure

A radian is the angle subtended by an arc of length r (radius)



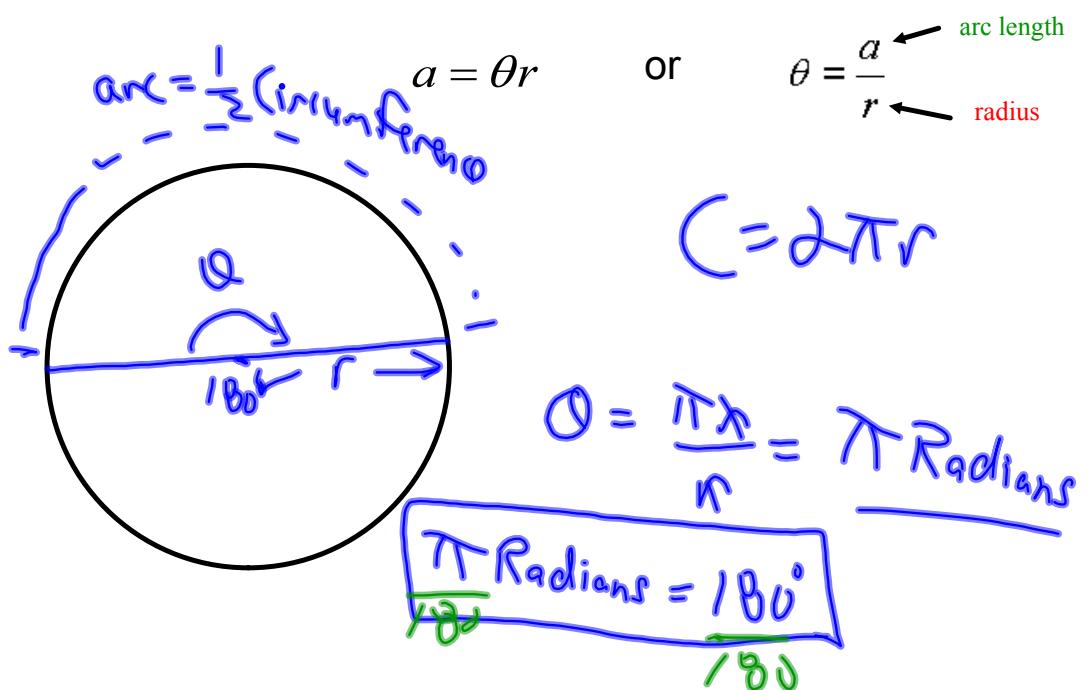
Use the above information to develop a formula to connect arc length, radius and the measure of an angle in radian measure...



$$a = \theta r$$

Radians \longleftrightarrow Degrees

Use this formula to develop a conversion factor between radians and degrees..



$$1^\circ = \frac{\pi}{180} \text{ radians}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

Strategies for converting between angle units...

Unitary Method

$$360^\circ = 2\pi$$

$$\begin{aligned} 1^\circ &= \frac{2\pi}{360} \\ &= \frac{\pi}{180} \end{aligned}$$

$$60^\circ = 60 \left(\frac{\pi}{180} \right) = \frac{60\pi}{180} = \frac{\pi}{3}$$

$$17^\circ = 17 \frac{\pi}{180}$$

Proportion Method

$$180^\circ = \pi$$

$$\frac{\pi}{180^\circ} \frac{60^\circ}{x} = \frac{x}{\pi}$$

$$x = \frac{60\pi}{180} = \frac{\pi}{3}$$

(1) In terms of π

$$50^\circ = \frac{50\pi}{180} = \frac{5\pi}{18}$$

(2) Decimal to nearest tenth

$$50^\circ = 0.9$$

Converting radians to degrees...

$$1 \text{ Rad} = \frac{180^\circ}{\pi}$$

Unit Analysis

$$\begin{aligned}\frac{5\pi}{4} &= \left(\frac{5\pi}{4}\right)\left(\frac{180^\circ}{\pi}\right) \\ &= \frac{5(180^\circ)}{4} \\ &= 225^\circ\end{aligned}$$

Why does $\left(\frac{180^\circ}{\pi}\right)$ have value 1?

$$30 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}$$

$$\frac{5\pi}{4} \text{ Rad} \cdot \frac{180^\circ}{\pi \text{ Rad}} = 225^\circ$$

$$\frac{3\pi}{7} = 77.1^\circ \quad \frac{3\pi}{7} \cdot \frac{180^\circ}{\pi}$$

$$\frac{3(180^\circ)}{7} =$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{3} = 60^\circ$$

$$\frac{\pi}{6} = 30^\circ$$

$$\frac{\pi}{2} = 90^\circ$$

$$\pi = 180^\circ$$

$$\frac{3\pi}{2} = 270^\circ$$

$$2\pi = 360^\circ$$

When each of the following angles is converted from degrees to radians the answer can be expressed as a multiple of π (note that it may be a fractional multiple). In each case state the multiple (e.g for an answer of $\frac{4\pi}{5}$ the multiple is $\frac{4}{5}$).

- a) 90°
- b) 360°
- c) 60°
- d) 45°
- e) 120°
- f) 15°
- g) 135°
- h) 270°

e) $\frac{2\pi}{3}$ f) $\frac{\pi}{12}$ g) $\frac{3\pi}{4}$

Convert each of the following angles from radians to degrees.

- a) $\frac{\pi}{2}$ radians
- b) $\frac{3\pi}{4}$ radians
- c) π radians
- d) $\frac{\pi}{6}$ radians
- e) 5π radians
- f) $\frac{4\pi}{5}$ radians
- g) $\frac{7\pi}{4}$ radians
- h) $\frac{\pi}{10}$ radians

a) 90°

e) 900°

b) 135°

f) 144°

c) 180°

g) 315°

d) 30°

h) 18°

Convert each of the following angles from degrees to radians giving your answer to 2 decimal places.

- a) 17° b) 49° c) 124° d) 200°

$$a) \frac{17\pi}{180} = 0.3$$

$$b) \frac{49\pi}{180} = 0.9$$

$$c) \frac{124\pi}{180} = 2.2$$

$$d) \frac{200\pi}{180} = 3.5$$

Convert each of the following angles from radians to degrees, giving your answer to 1 decimal place.

- a) 0.6 radians b) 2.1 radians c) 3.14 radians d) 1 radian

$$\text{a)} \frac{0.6 \text{ Radians}}{\pi \text{ Radians}} \times 180^\circ$$

$$= 34.4^\circ$$

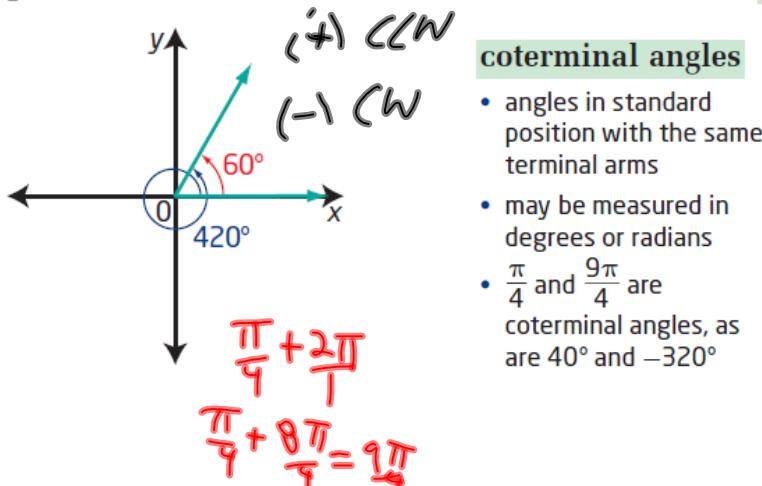
$$\text{b)} 120.3^\circ$$

$$\text{c)} 179.9^\circ$$

$$\text{d)} \underline{57.3^\circ}$$

Coterminal Angles

When you sketch an angle of 60° and an angle of 420° in standard position, the terminal arms coincide. These are **coterminal angles**.



Strategy for finding coterminal angles in either radians or degrees?

$$\left. \begin{array}{l} \text{Degrees} \\ \theta + 360^\circ k, k \in \mathbb{Z} \end{array} \right\} \quad \left. \begin{array}{l} \text{Radians} \\ \theta + 2\pi k, k \in \mathbb{Z} \end{array} \right.$$

$$\theta = \frac{7\pi}{8}$$

4 co-terminal angles ...
2 Positive & 2 Negative

$$\theta_1 = \frac{7\pi}{8} + 2\pi = \frac{7\pi}{8} + \frac{16\pi}{8} = \frac{23\pi}{8}$$

$$\theta_2 = \frac{39\pi}{8}$$

$$\theta_3 = \frac{7\pi}{8} - \frac{16\pi}{8} = -\frac{9\pi}{8}$$

$$\theta_4 = -\frac{25\pi}{8}$$

For each angle in standard position, determine one positive and one negative angle measure that is coterminal with it.

a) 270°

$$\theta = 430^\circ$$

$$\theta = -90^\circ$$

b) $-\frac{5\pi}{4}$

$$\theta = \frac{3\pi}{4}$$

$$\theta = -\frac{13\pi}{4}$$

c) 740°

$$\theta = 1100^\circ, 380^\circ, 20^\circ$$

$$\theta = -340^\circ$$

$$\theta = 1738429^\circ$$

$$\frac{\div 360^\circ}{}$$

$$\begin{array}{r} 4828 \\ - 4828 \\ \hline \end{array}$$

$$0. \dots \times 360$$

$$\begin{array}{r} -349 \\ -360^\circ \\ \hline -11^\circ \end{array}$$

Principal Angle

$$\theta = -11^\circ$$

$$\frac{1347\pi}{6}$$

$$1347 \div 6 = 224.5$$

$$\left(\frac{1344\pi}{6}\right) + \frac{3\pi}{6}$$

$$224 \times 6 = \underline{\underline{1344}}$$

$$224\pi + \frac{3\pi}{6}$$

Back
Siney

$$\theta = \frac{\pi}{2} - 2\pi = \frac{\pi}{2} - \frac{4\pi}{2} = -\frac{3\pi}{2}$$

$$\frac{1347\pi}{6} - 999\left(\frac{12\pi}{6}\right)$$

Coterminal Angles in General Form

Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation from the terminal arm, you arrive back at the same terminal arm. Angles coterminal with any angle θ can be described using the expression

$$\theta \pm (360^\circ)n \text{ or } \theta \pm 2\pi n,$$

where n is a natural number. This way of expressing an answer is called the **general form**.

general form

- an expression containing parameters that can be given specific values to generate any answer that satisfies the given information or situation
- represents all possible cases

Let's use the following two angles...

$$\theta = 70^\circ$$

$$\theta = \frac{5\pi}{6}$$

$$\left. \begin{array}{l} \theta = 70^\circ + 360K, \\ \theta = \frac{5\pi}{6} + 2\pi K, \end{array} \right\} \begin{array}{l} K \in \mathbb{I} \\ K \in \mathbb{C} \end{array}$$