Determine a negative angle co-terminal with each of the following angles:

1) $476895°$

1324.708...
↑ Full Rotations

$0.708... \times 360°$

$= 255°$

$-360°$

$-105°$

2) $\frac{35784\pi}{5}$

$\frac{35784\pi}{5} \div \frac{2\pi}{1}$

$\frac{35784\pi}{5} \cdot \frac{1}{2\pi}$

3578.4 Rotations

0.4 of a Rotation

$0.4 \times 2\pi = 2.513$ Rad

-2π

-3.8 Rad

$\frac{4}{10} \times 2\pi = \frac{8\pi}{10}$

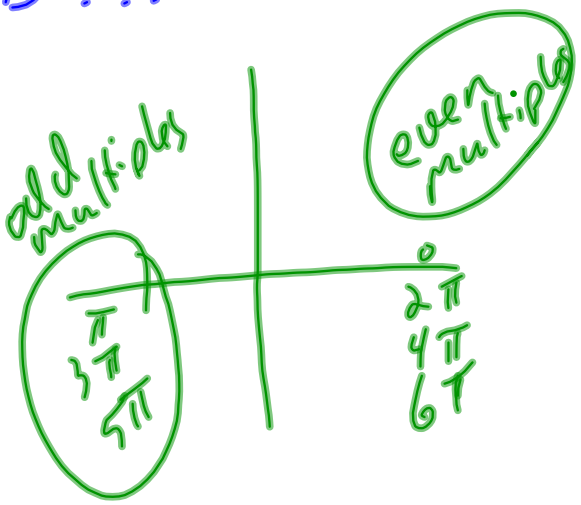
$= \frac{4\pi}{5} - \frac{2\pi}{1}$

$= \frac{4\pi}{5} - \frac{10\pi}{5} = -\frac{6\pi}{5}$

2) $\dfrac{35784\pi}{5}$ ← Plan B . . .

$\frac{35785}{5}\pi - \frac{\pi}{5}$

$7157\pi - \frac{\pi}{5}$



$\pi - \frac{\pi}{5}$

$= \frac{4\pi}{5} - \frac{2\pi}{1}$

$= -\frac{6\pi}{5}$

or

$-\pi - \frac{\pi}{5}$

$-\frac{6\pi}{5}$

ex. $\frac{139427\pi}{4}$

$\frac{139428\pi}{4} - \frac{\pi}{4}$

$34857\pi - \frac{\pi}{4}$

odd

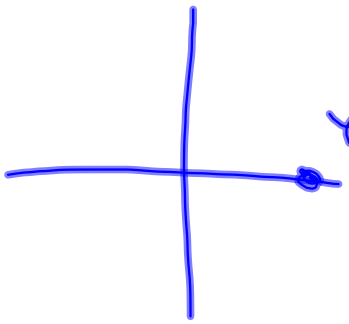
$-\pi - \frac{\pi}{4}$

$= -\frac{5\pi}{4}$

$\frac{139424\pi}{4} + \frac{3\pi}{4}$

$34856\pi + \frac{3\pi}{4}$

even



$-\frac{2\pi}{1} + \frac{3\pi}{4}$

$= -\frac{5\pi}{4}$

$$\frac{2985\pi}{7}$$

$$= \frac{2982\pi}{7} + \frac{3\pi}{7}$$

$$= 426\pi + \frac{3\pi}{7}$$

even

$$-\frac{2\pi}{1} + \frac{3\pi}{7}$$

$$= -\frac{11\pi}{7}$$

Coterminal Angles in General Form

Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation from the terminal arm, you arrive back at the same terminal arm. Angles coterminal with any angle θ can be described using the expression

$\theta \pm (360°)n$ or $\theta \pm 2\pi n$, $\theta + 360°k, k \in I$

where n is a natural number. This way of expressing an answer is called the **general form**. $\theta + 2\pi k, k \in I$

general form

- an expression containing parameters that can be given specific values to generate any answer that satisfies the given information or situation
- represents all possible cases

Let's use the following two angles...

$\theta = 70°$ $\theta = \frac{5\pi}{6}$

$= 70° + 360°k, k \in I$ $= \frac{5\pi}{6} \pm 2\pi k, k \in N$

$I \Rightarrow \mathbb{Z}$

Integers

What if we are given a restricted domain?

$\theta = 70° \text{ , } -720° \le \theta \le 1080°$

$\theta = 70° + 360°K, K \in I$

$\theta_1 = 70° + 360°(1) = 430°$

$\theta_2 = 70° + 360°(2) = 790°$

$\theta_3 = 70° + 360°(3) = 1150°$

$\theta_4 = 70° + 360°(-1) = -290°$

$\theta_5 = 70° + 360°(-2) = -650°$

$\theta_6 = 70° + 360°(-3) = -1010°$

$\theta = \frac{5\pi}{6} \text{ , } -2\pi \le \theta \le 8\pi$

$\theta = \frac{5\pi}{6} + \frac{2\pi}{1} = \frac{17\pi}{6}$

$\theta = \frac{17\pi}{6} + \frac{2\pi}{1} = \frac{29\pi}{6}$

$\theta = \frac{29\pi}{6} + \frac{2\pi}{1} = \frac{41\pi}{6}$

$\theta = \frac{41\pi}{6} + \frac{2\pi}{1} = \frac{53\pi}{6}$

$\theta = \frac{5\pi}{6} - \frac{2\pi}{1} = -\frac{7\pi}{6}$

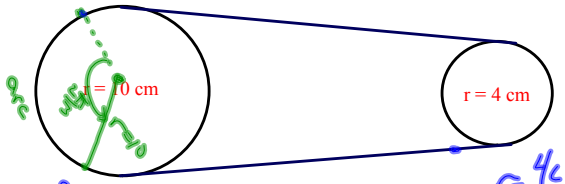
$\theta = -\frac{7\pi}{6} - 2\pi = -\frac{19\pi}{6}$

Applying our knowledge of rotations and radians... $\theta = \frac{a}{r}$

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger whell rotate?



$a = \theta r$

$a = \frac{2\pi}{3}(10)$

$a = \frac{20\pi}{3}$ cm

$r = 4$ cm

$a = \frac{20\pi}{3}$ cm

$\theta = \frac{\frac{20\pi}{3}\text{ cm}}{4\text{ cm}}$

Larger Pulley ($r = 10$ cm)

$\theta = \frac{2\pi}{3}$

$\frac{10}{4} = 2.5$ times

$\times 2.5$

Smaller Pulley ($r = 4$ cm)

$\theta = \frac{5\pi}{3}$

$\theta = \frac{20\pi}{3} \cdot \frac{1}{4}$

$\theta = \frac{20\pi}{12} = \frac{5\pi}{3}$

b) Larger Wheel $\Rightarrow$ 3 Rev.

Smaller Wheel $\Rightarrow 3 \times 2.5 = 7.5$ Rev's

1 Rev $= 2\pi$ Rad

$7.5 \text{ Rev} \times \frac{2\pi \text{ Rad}}{1 \text{ Rev}} = 15\pi$ Rad

c) Smaller: $\theta = \frac{-15\pi}{4}$

Larger: $\theta = \frac{-15\pi}{4} \div 2.5$

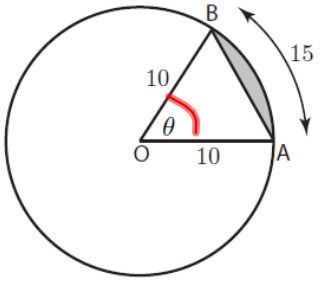
$\theta = \frac{-15\pi}{4} \div \frac{5}{2}$

$\theta = \frac{-15\pi}{4} \times \frac{2}{5}$

$\theta = \frac{-30\pi}{20} = \frac{-3\pi}{2}$

Example

Refer to Figure 8. Suppose we have a circle of radius 10cm and an arc of length 15cm. Suppose we want to find (a) the angle θ, (b) the area of the sector OAB, (c) the area of the minor segment (shaded).



Area of Sector

$A = \left(\frac{\theta}{360}\right)\pi r^2$

Figure 8. The shaded area is called the minor segment.

$A = \frac{1}{2}ab\sin\theta$

Practice Problems...

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#3, 4, 5, 6, 7, 9, 11, 12, 13