Sketch $f(x)$

①

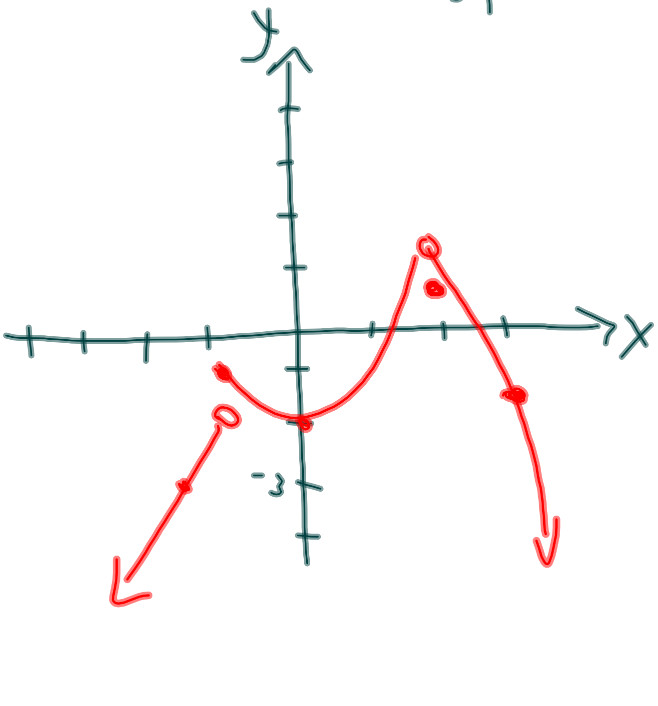
x	y
-1	-2
-2	-3

$$f(x)=\begin{cases} x-1 \quad , & if \;\; x<-1 \\ x^2-2 \quad , & if \;\; -1\le x<2 \\ 1 \quad , & if \;\; x=2 \\ -(x-1)^2+3 \quad , & if \;\; x>2 \end{cases}$$

② V(0,-2) ③ (2,1) ④ V(1,3) ← <u>Not in domain</u> !!

x	y
-1	-1
2	2

x	y
2	2
3	-1



6. $y = x^2 \Longrightarrow$ original graph

$y = x^2$

x	y
4	16

$+3$

Now

x	y
4	19

$(4, 19)$ vertical translation

7. $y = x^2 \Longrightarrow$

x	y
4	16

New

Right 1

x	y
5	16

$(5, 16)$

Check-Up...

Copy and complete the table.

Pg. 13
#8

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$
H & V	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$

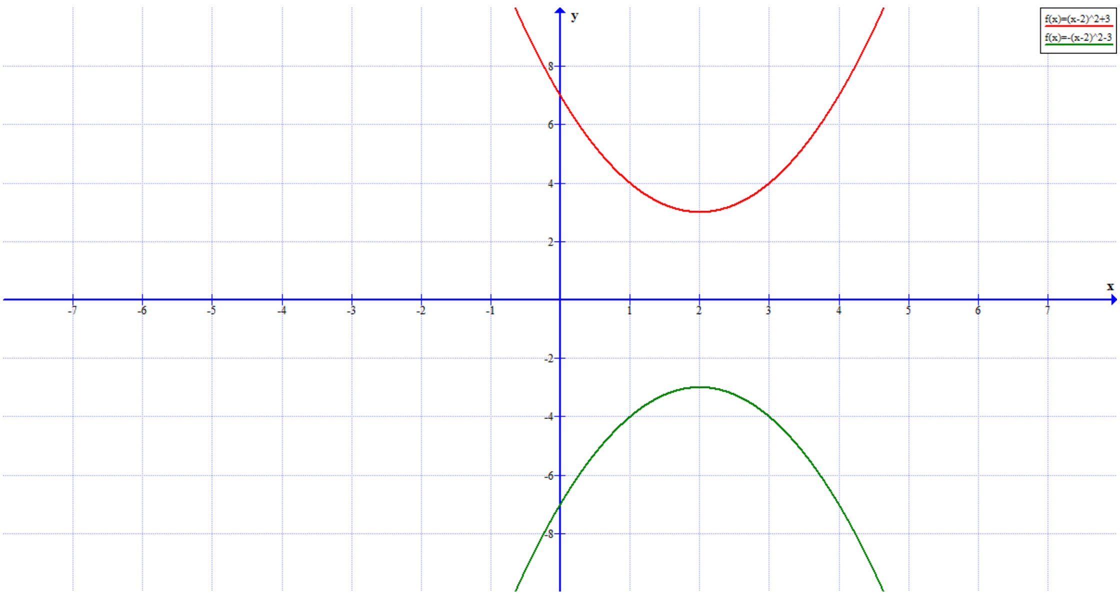
Reflections and Stretches

Focus on...

- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

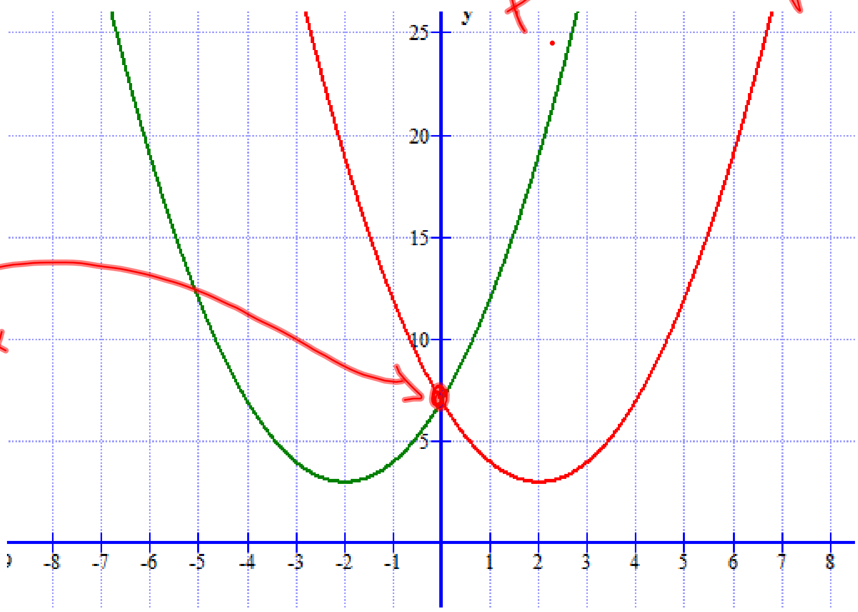


Mapping: $(x, y) \rightarrow (x, -y)$

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

$f(-x)$ $f(x)$

Invariant Point



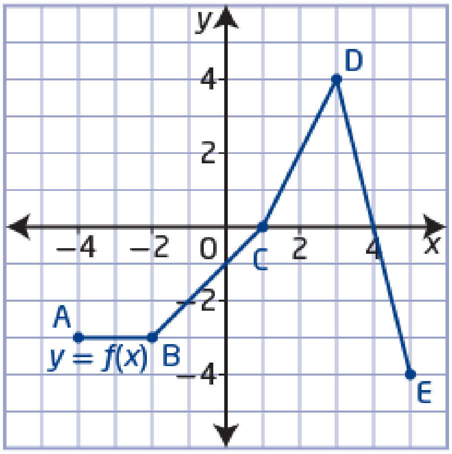
Mapping: $(x, y) \rightarrow (-x, y)$

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

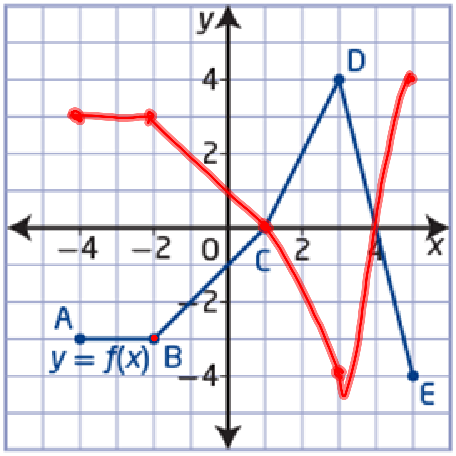
b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?

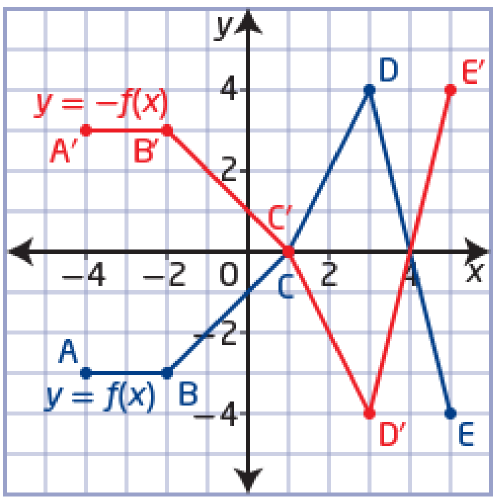


Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below

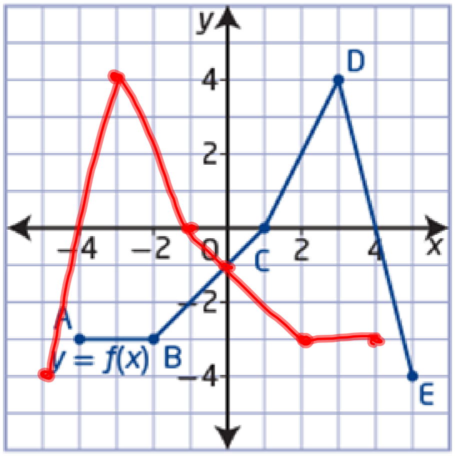


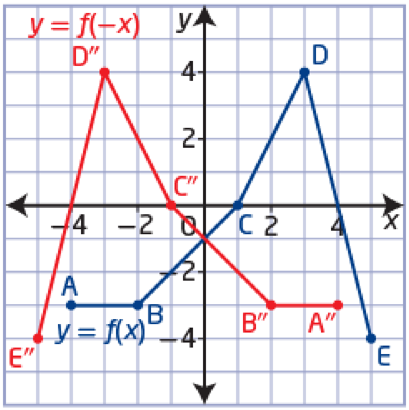


Remember...

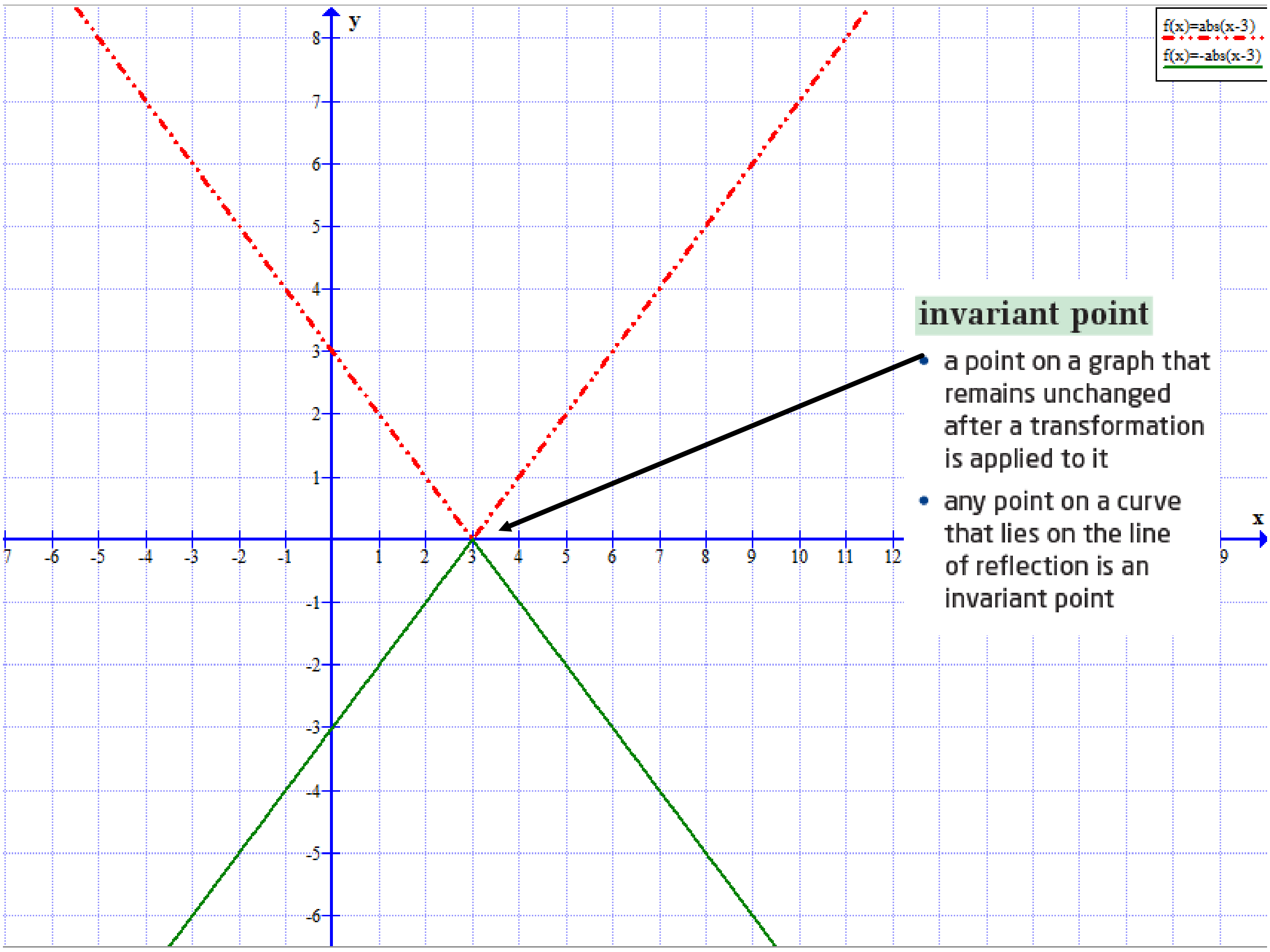
- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below





Invariant Point



stretch compression

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor

- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

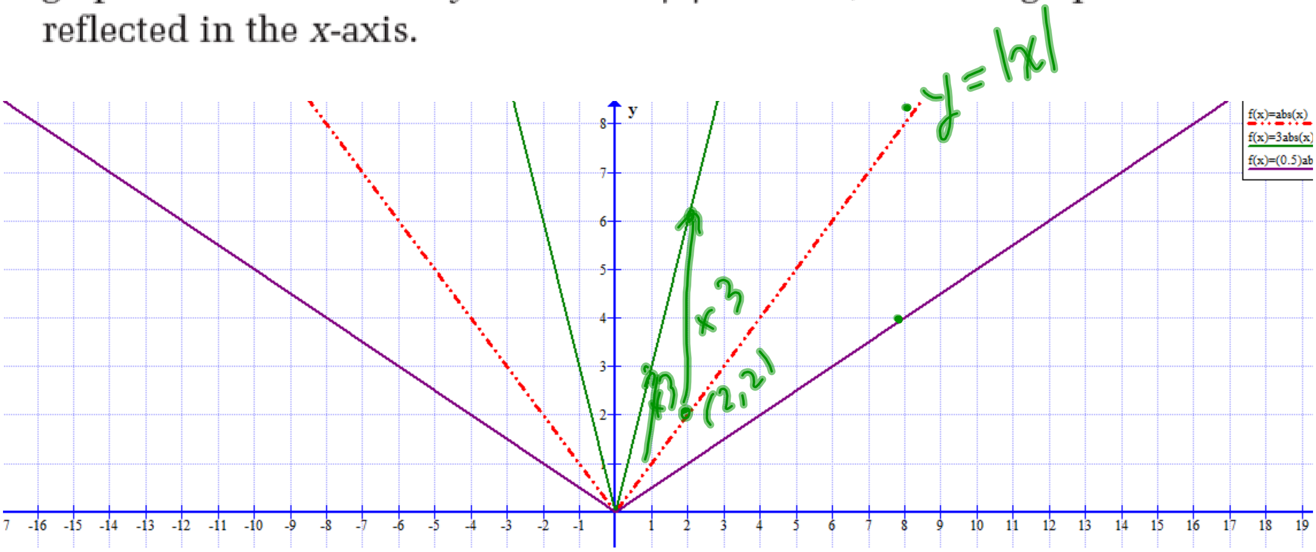
Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Vertical Stretch or Compression...

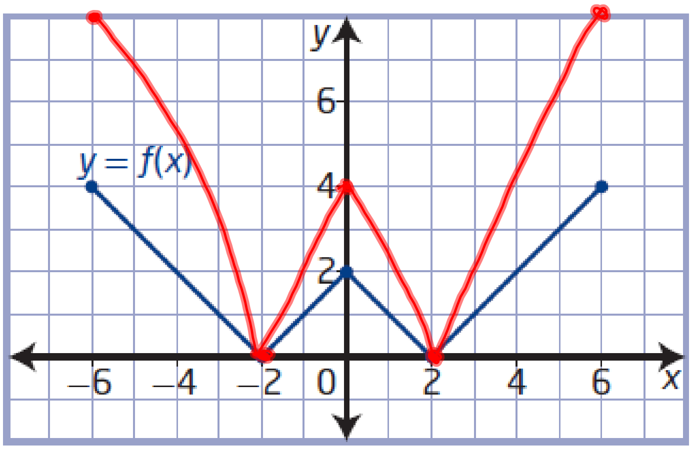
- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.



$y = |x|$ $\qquad$ $y = 3|x|$ $\qquad$ $y = \frac{1}{2}|x|$

$y = af(x)$

Mapping: $(x, y) \rightarrow (x, ay)$

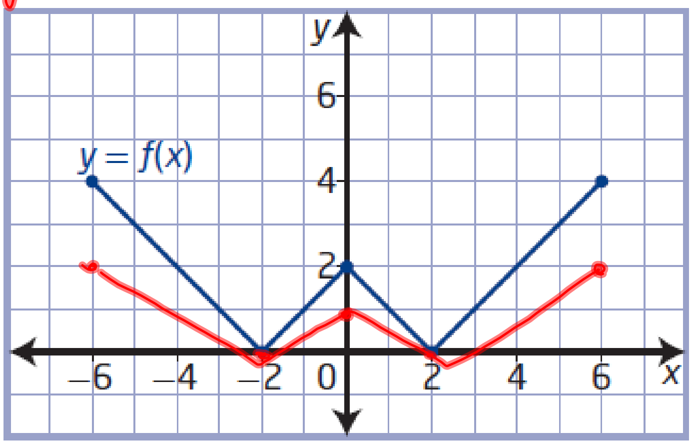


Sketch each of the following:

a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$

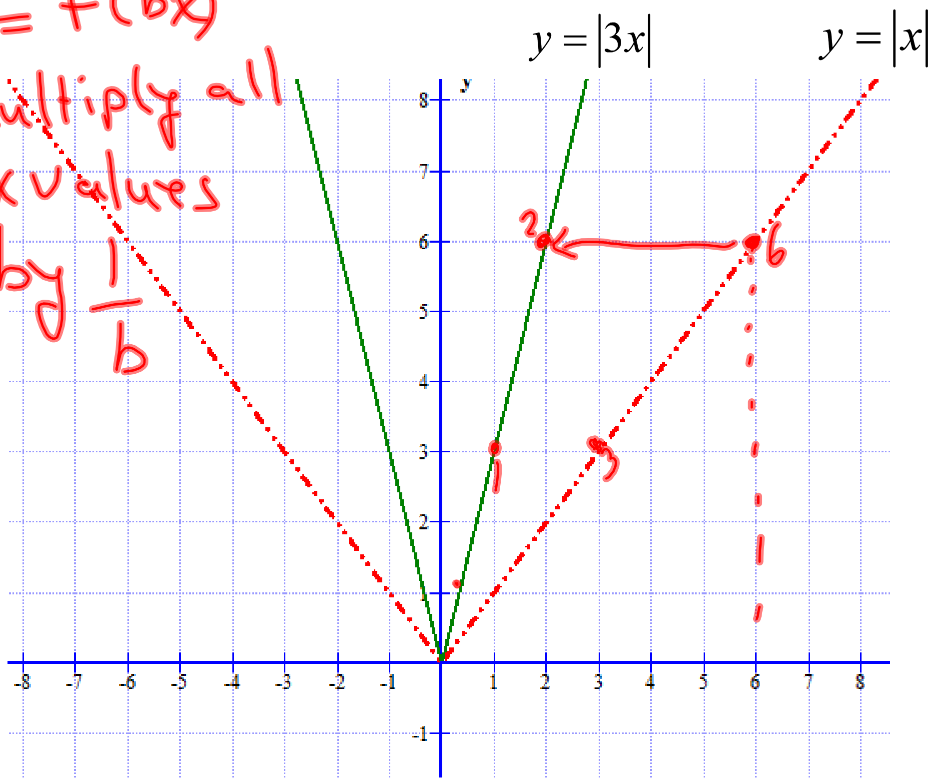
$y = \frac{1}{2}f(x)$



Horizontal Stretch or Compression...

$y = f(bx)$

Multiply all x values by $\frac{1}{b}$

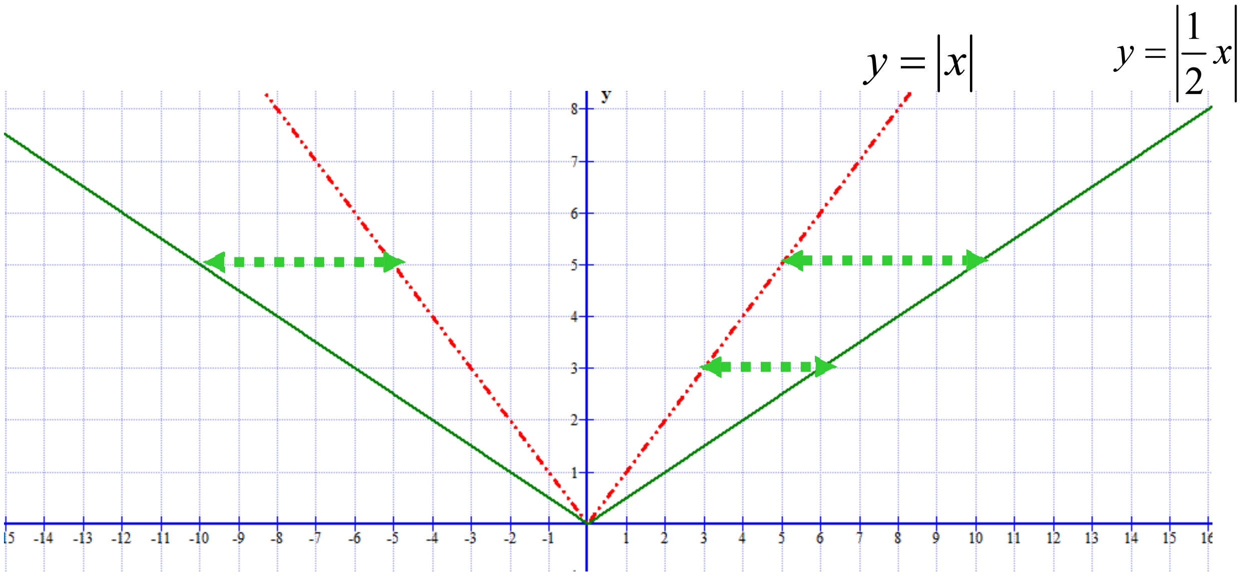


$y = f(bx)$

$(x, y) \to (\frac{1}{b}x, y)$

$y = f(\frac{1}{5}x)$

$(x, y) \to (5x, y)$


$y = |x|$
$y = \left|\frac{1}{2}x\right|$
y

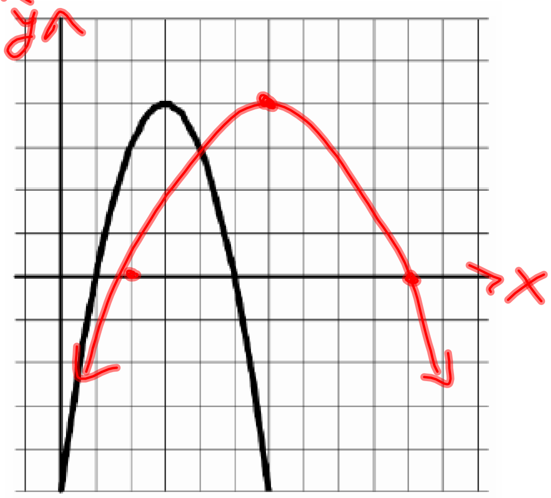
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = f(-2x)$

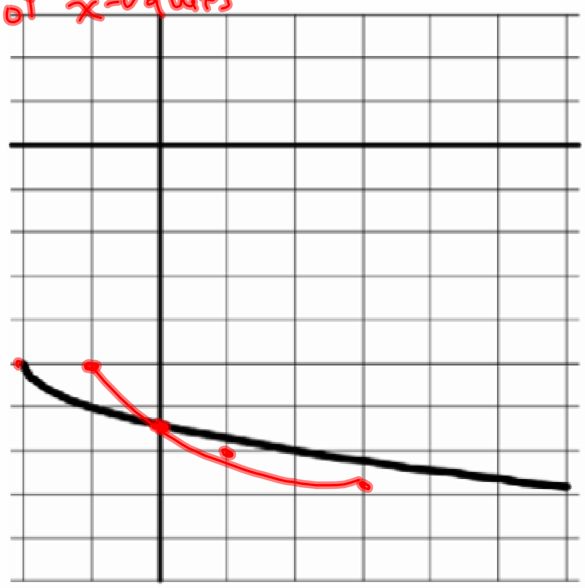
Example 1: Apply $f\left(\frac{1}{2}x\right)$ to the graph.

⇒ Double x-values



Apply f(2x) to the given graph.

$\frac{1}{2}$ of x-values



$$y = -3f(-2x) + 7$$

$$-\left(\frac{1}{2}\right)x$$

$$(x, y) \longrightarrow \left(-\frac{1}{2}x, -3y + 7\right) \qquad -\frac{1}{2}x$$

⇒ Reflected in x & y axis

⇒ Stretch vertically by a factor of 3

⇒ " horizontally " " " $\frac{1}{2}$

⇒ Shift up 7