

Warm-Up...

Given that $(-2, 5)$ is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$$(-2, 5) \rightarrow ??$$

$$(1) y = 3f(x)$$

$$(-2, 5) \rightarrow (-2, 15)$$

$$(x, y) \rightarrow (x, 3y)$$

$$(-2, 5) \rightarrow (-2, 15)$$

$$(2) y = f\left(-\frac{1}{3}x\right)$$

$$(-2, 5) \rightarrow (6, 5)$$

$$(x, y) \rightarrow (-3x, y)$$

$$\rightarrow (6, 5)$$

$$(3) y = 4f\left[\frac{1}{2}(x+5)\right] - 3$$

$$(x, y) \rightarrow (2x - 5, 4y - 3)$$

$$(-2, 5) \rightarrow (-4 - 5, 20 - 3)$$

$$\rightarrow (-9, 17)$$

$$(4) y - 5 = -2f(-2x + 6)$$

$$y = -2f[-2(x-3)] + 5$$

$$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$$

$$(-2, 5) \rightarrow \left(-\frac{1}{2}(-2) + 3, -2(5) + 5\right)$$

$$\rightarrow (4, -5)$$

Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + c$	shift $f(x)$ up c units
$f(x) - c$	shift $f(x)$ down c units
$f(x + c)$	shift $f(x)$ left c units
$f(x - c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$
	When $c > 1$ – vertical stretching of $f(x)$ Multiply the y values by c
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$
	When $c > 1$ – horizontal shrinking of $f(x)$ Divide the x values by c

$$y = f(x) \longrightarrow y = af(b(x - c)) + d$$

Mapping Rule: $(x, y) \rightarrow \left(\frac{1}{b}x + c, ay + d\right)$

✓

Important note for sketching...

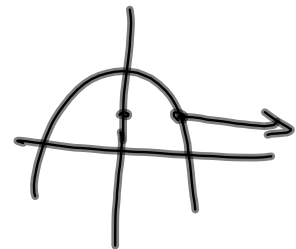
Transformations should be applied in following order:

- 1. Reflections**
- 2. Stretches**
- 3. Translations**

Remember...RST

The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.



a) y-axis

d) 3

g) 10

b) $\frac{1}{4}$

e) x-axis

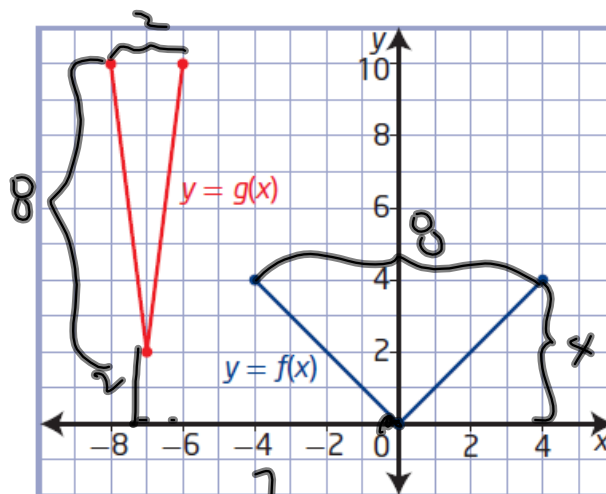
c) x-axis

*f) 4

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.

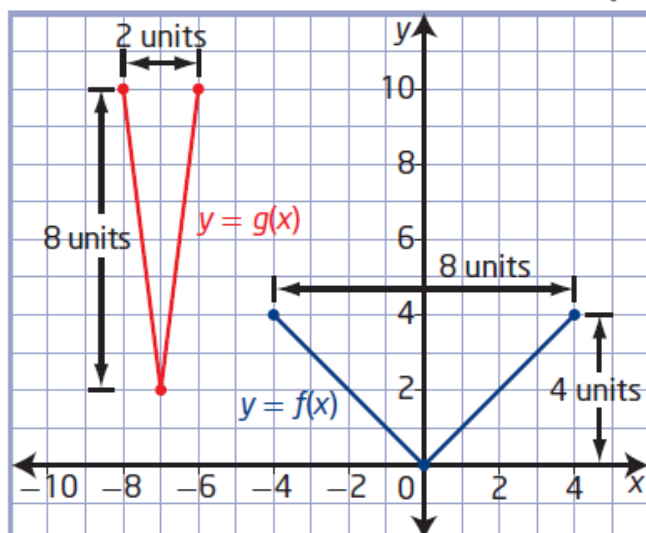
$$y = 2f\left[\frac{1}{4}(x+7)\right] + 2$$



Solution

The equation of the transformed function is $g(x) = 2f\left(\frac{1}{4}(x+7)\right) + 2$.

$(x, y) \rightarrow \left(\frac{1}{4}x - 7, 2y + 2\right)$
 $(-4, 4) \rightarrow (-8, 10)$



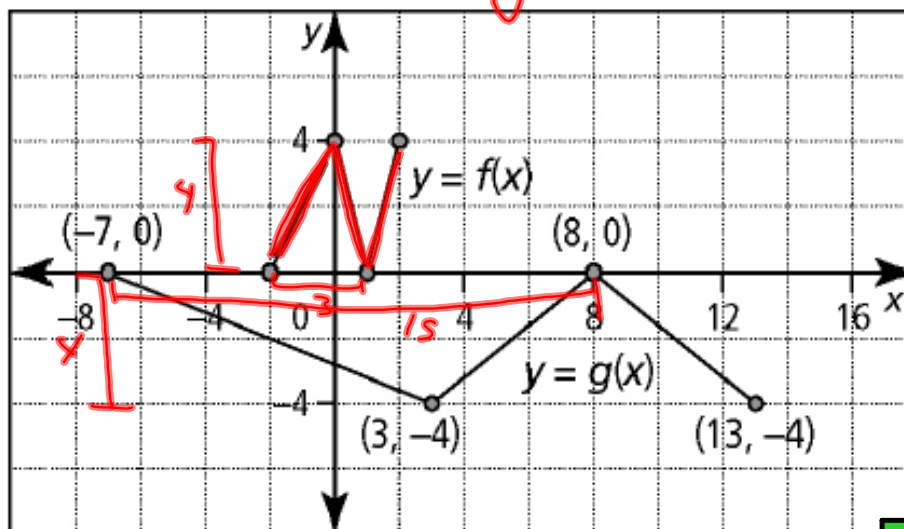
How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$.

Determine the equation of $g(x)$ in the form

$y = af(b(x - h)) + k$.

$$y = -1f\left[\frac{1}{5}(x-3)\right]$$



$$y = -f\left(\frac{1}{5}(x-3)\right)$$

Example...

The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

$$y = 2x^2 + x + 1$$

$$y = 2\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) + \frac{1}{16} - \frac{1}{8}$$

$$y = 2\left(x + \frac{1}{4}\right)^2 + \frac{7}{8}$$

$$y = 4\left[3\left(x - \frac{7}{4}\right)\right]^2 - \frac{25}{8}$$

$$\frac{1}{4} - \frac{2}{1} = \frac{1}{4} - \frac{8}{4} = -\frac{7}{4}$$

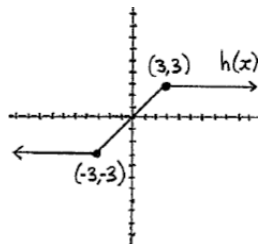
$$\frac{7}{8} - 4 = \frac{7}{8} - \frac{32}{8} = -\frac{25}{8}$$

$$(x^2) \quad (x + \frac{1}{4} - 2)$$

$$(x-2)$$

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

Warm-Up...



$$h\left(\frac{x}{3}\right) = h\left(\frac{1}{3}x\right)$$

Given the graph of $h(x)$ above, match the following four functions with their graphs.

29.) $3h(x)$

A

30.) $\frac{1}{3}h(x)$

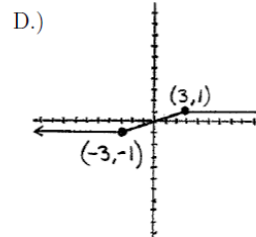
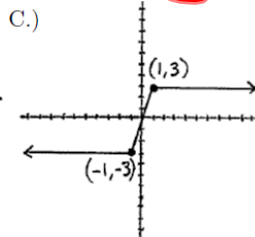
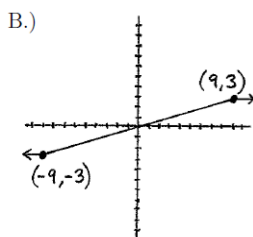
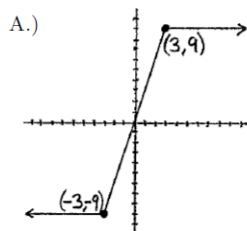
D

31.) $h(3x)$

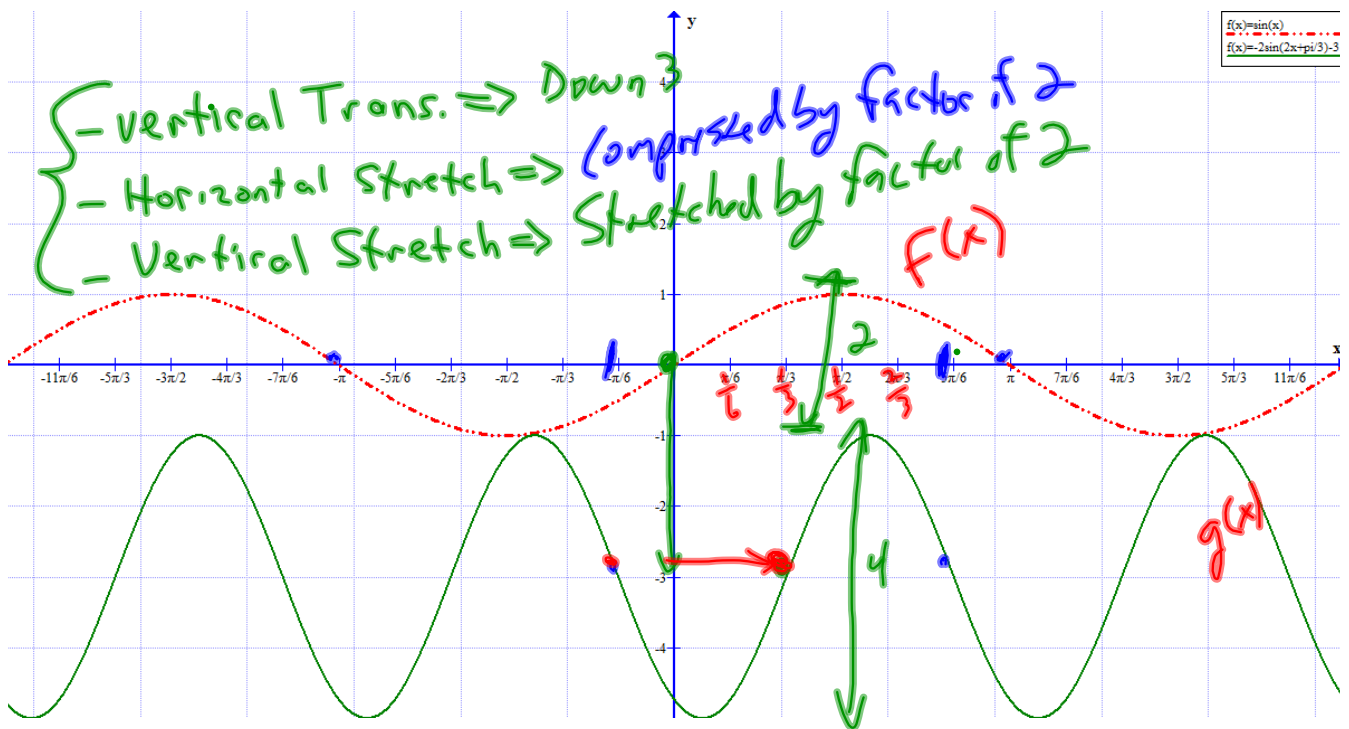
C

32.) $h\left(\frac{x}{3}\right)$

B



How did the graph of $y=f(x)$ become the green graph, $y = g(x)$??



$$y = 2f\left[2\left(x - \frac{1}{3}\right)\right] - 3$$

or

$$y = -2f\left[2\left(x + \frac{1}{6}\right)\right] - 3$$

Practice problems...

Sec 1.2

Page 28 - 31

#5, 6, 7, 8, 9, 14, C4

Practice Problems...

Sec 1.3

Pages 39 - 41

#3, 4, 6, 7, 8, 10, 13, 14