Warm Up

Review of laws of logarithms…

Solve the following equation: $\frac{3^{x-1}}{5 * 2^{3x}} = 6^{1-2x}$

$\ln\left(\frac{3^{x-1}}{5 \cdot 2^{3x}}\right) = \ln(6)^{1-2x}$

$(x-1)\ln 3 - \ln 5 - 3x\ln 2 = (1-2x)\ln 6$

$x\ln 3 - \ln 3 - \ln 5 - x3\ln 2 = \ln 6 - x2\ln 6$

$x(\ln 3 - 3\ln 2 + 2\ln 6) = \ln 6 + \ln 3 + \ln 5$

$x = \frac{(\ln 6 + \ln 3 + \ln 5)}{(\ln 3 - 3\ln 2 + 2\ln 6)} \doteq 1.73$

$7^x = 12^{x+5}$

$\ln 7^x = \ln 12^{x+5}$

$x\ln 7 = (x+5)\ln 12$

$x\ln 7 = x\ln 12 + 5\ln 12$

$x\ln 7 - x\ln 12 = 5\ln 12$

$x(\ln 7 - \ln 12) = 5\ln 12$

$x = \frac{5\ln 12}{\ln 7 - \ln 12}$

$x \doteq -23.05$

Derivative Rules:

① $d(b^u) = b^u(\ln b)\,du \quad (b \in \mathbb{R})$

② $d(e^u) = e^u \cdot du$

① $d(\log_b u) = \frac{1}{u \ln b} \cdot du$

② $d(\ln u) = \frac{1}{u}\,du$

Logarithmic Differentiation

A differentiation process that requires taking the logarithm of both sides before differentiating.

This process will be used in TWO circumstances:

I. Simplifying messy products and quotients

What would it involve to differentiate the following?

$$y=\frac{\left(x^2-1\right)^5\sqrt{2x+9}\left(5x^3+2\right)^8}{(10x-1)\sqrt{5-x^7}}$$

- Quotient rule, multiple product rules and chain rules…

This would be possible but it would be easier to differentiate a group of terms added and subtracted rather than multiplied and divided

Laws of logarithms will do exactly that...turn this mess into a addition and subtraction of terms.

$$\ln y=\ln\left[\frac{\left(x^2-1\right)^5\sqrt{2x+9}\left(5x^3+2\right)^8}{(10x-1)\sqrt{5-x^7}}\right]$$

$$\frac{dy}{y}$$

$$\ln y = 5\ln(x^2-1)+\frac{1}{2}\ln(2x+9)+8\ln(5x^3+2)-\ln(10x-1)-\frac{1}{2}\ln(5-x^7)$$

$$\frac{1}{y}\boxed{\frac{dy}{dx}}=\left[5\left(\frac{2x}{x^2-1}\right)+\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{2x+9}\right)+8\left(\frac{15x^2}{5x^3+2}\right)-\left(\frac{10}{10x-1}\right)-\frac{1}{2}\left(\frac{-7x^6}{5-x^7}\right)\right]y$$

$$\frac{dy}{dx}=\left[\frac{10x}{x^2-1}+\frac{1}{2x+9}+\frac{120x^2}{5x^3+2}-\frac{10}{10x-1}+\frac{7x^6}{2(5-x^7)}\right]\frac{\left(x^2-1\right)^5\sqrt{2x+9}\left(5x^3+2\right)^8}{(10x-1)\sqrt{5-x^7}}$$

Example:

Differentiate: $\ln y = \ln\left[\frac{(3-2x^5)^6(x^5-1)}{(2x+7)^8(x^{-5}+2)^4}\right]$

$$\ln y = 6\ln(3-2x^5) + \ln(x^5-1) - 8\ln(2x+7) - 4\ln(x^{-5}+2)$$

$$\frac{1}{\cancel{y}}\frac{dy}{dx} = \left[6\left(\frac{-10x^4}{3-2x^5}\right) + \left(\frac{5x^4}{x^5-1}\right) - 8\left(\frac{2}{2x+7}\right) - 4\left(\frac{-5x^{-6}}{x^{-5}+2}\right)\right]\left[\frac{(3-2x^5)^6(x^5-1)}{(2x+7)^8(x^{-5}+2)^4}\right]$$

II. Base and exponent both variables

Have a look at this example:

$$y = x^{x^5}$$

- Does not fit either the power rule or the rules for an exponential function

...What can be done to help this crazy situation??

Of Course...take the logarithm of both sides!!

$$y = x^{x^5}$$

$$\ln y = \ln x^{x^5}$$

$$\ln y = \underbrace{x^5}\,\underbrace{\ln x}$$

$$(y)\ \frac{1}{\cancel{y}}\frac{dy}{dx} = \left(5x^4 \ln x + x^5 \frac{1}{x}\right)(y)$$

$$\frac{dy}{dx} = \left(5x^4 \ln x + x^4\right) x^{x^5}$$

Example:

Differentiate: $y = \left(\ln x^5\right)^{\cos x}$

$$\ln y = \ln\left[\left(\ln x^5\right)\right]^{\cos x}$$

$$\ln y = \cos x \ln\left(\ln x^5\right)$$

$$(y)\ \frac{1}{y}\frac{dy}{dx} = \left[(-\sin x)\ln\left(\ln x^5\right) + \cos x\left(\frac{1}{\ln x^5}\right)\left(\frac{5x^4}{x^5}\right)\right]\left(\ln x^5\right)^{\cos x}$$

Practice Questions...

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