Antiderivatives

Differentiate

$$f(x) \qquad\qquad f'(x)$$

Antidifferentiate

Definition A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example:

antidifferentiate

$$F(x)=\frac{x^2}{2} \qquad\qquad F'(x)=x$$

diff.

What about $F(x)=\frac{x^2}{2}+2$?

1 Theorem If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

General antiderivatives are considered a family of curves...

Here is an example of a family of general antiderivatives:

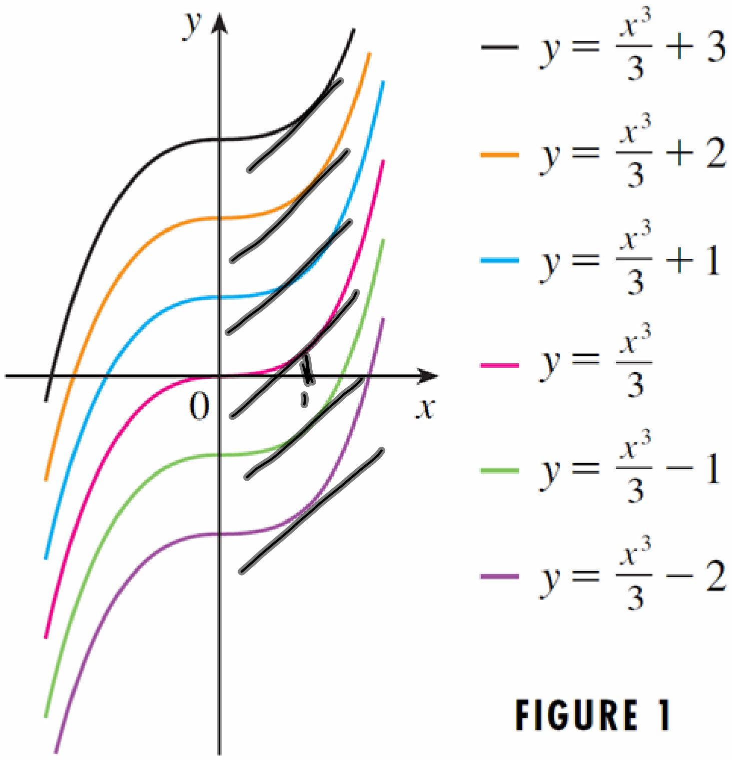

FIGURE 1

Notice the slopes of the tangents on each curve at the same *x*-coordinate.

Antidifferentiation Rules...

Constants:

Determine the antiderivative of any constant

$f'(x) = 6 \Rightarrow f(x) = 6x + C$

Rule:

$$f(x) = k \quad \Rightarrow \quad F(x) = kx + C$$

Power Law:

How will we put the power rule in reverse?

ie. If $f'(x) = 6x^2$ what is $f(x)$?

$f(x) = 2x^3 + C$

Flip your brain into reverse...what will be the rule used to antidifferentiate power rules?

Rule:

$$f(x) = kx^n \Rightarrow F(x) = \frac{k}{n+1} x^{n+1} + C$$

Add one to the exponent and divide by this NEW exponent

Determine the general antiderivative of each of the following...

$2x^{-2}$

(1) $f(x) = x^5 - 2x^4 - 3x^3 + \frac{2}{x^2} + 5x^{-3} + 5$

$F(x) = \frac{1}{6}x^6 - \frac{2}{5}x^5 - \frac{3}{4}x^4 - 2x^{-1} - \frac{5}{2}x^{-2} + 5x + C$

$3x^{1/2} - \frac{2}{5}x^{-4} + x^{7/5} - 6x^{-3/2} + e^2$

$\frac{x^{1/2}}{x^2}$

(2) $f(x) = 3\sqrt{x} - \frac{2}{5x^4} + \sqrt[5]{x^7} - \frac{6\sqrt{x}}{x^2} + e^2$

$F(x) = 2x^{3/2} + \frac{2}{15}x^{-3} + \frac{5}{12}x^{12/5} + 12x^{-1/2} + e^2x + C$

- ✓ constants
- ✓ power rules
- logarithmic functions
- trigonometric functions
- exponential functions
- inverse trigonometric functions
- chain rules

Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$y = 6x^3$

$y' = 6(3x^2)$

$$f'(x) = n[g(x)]^{n-1}\, g'(x)$$

$u^n \cdot du$

Examples:

$u^n \cdot du$

(1) $F(x) = \frac{1}{8}(8x-5)^9\,(8)$

$u^n \cdot du \Rightarrow \frac{1}{n+1}u^{n+1}$

$F(x) = \frac{1}{8}\left[\frac{1}{10}(8x-5)^{10}\right] + C$

$F(x) = \frac{1}{80}(8x-5)^{10} + C$

$F'(x) = \frac{10}{80}(8x-5)^9(8)$

(2) $F(x) = x^3\sqrt{1-5x^4}$

$F(x) = \frac{-1}{20}(1-5x^4)^{1/2}(-20x^3)$

$u^n \cdot du$

$F(x) = \frac{-1}{20}\left[\frac{2}{3}(1-5x^4)^{3/2}\right] + C$

$F(x) = -\frac{1}{30}(1-5x^4)^{3/2} + C$

(3) $F(x) = \dfrac{x^3}{\sqrt[3]{1-x^4}}$

(4) $F(x) = \dfrac{(5-3x^{-4})^5}{2x^3}$

Practice Set...

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Red Textbook

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Warm Up

Determine the general antiderivative for the following:

$$F(x) = -2x^5 + 7\sqrt{x} - \frac{3}{4x^3} + 5\sqrt[4]{x^3} - \sec^2 x + 2\pi^3$$

$$F(x) = -2x^5 + 7x^{1/2} - \frac{3}{4}x^{-3} + 5x^{3/4} - \sec^2 x + 2\pi^3$$

$$f(x) = \frac{-1}{3}x^6 + \frac{14}{3}x^{3/2} + \frac{3}{8}x^{-2} + \frac{20}{7}x^{7/4} - \tan x + 2\pi^3 x + C$$

$$\text{Position} \underset{\text{Antidifferentiate}}{\overset{\text{Differentiate}}{\rightleftarrows}} \text{Velocity} \underset{\text{Antidifferentiate}}{\overset{\text{Differentiate}}{\rightleftarrows}} \text{Acceleration}$$

Example 1:
A golfer on the moon (where gravitational acceleration equals 1.67 m/sec^2)
hits a ball whose initial velocity in the vertical direction is 30 meters per second.
What is the maximum height the ball reaches?

269.46 m

Velocity = 0 m/s

$\boxed{a=-1.67}$

Initial conditions

$V=-1.67t+C$ → $t=0$, $v=30$ m/s

$h=0$ m

Substitute initial condition...

$30=-1.67(0)+C$

$C=30$

$\boxed{V=-1.67t+30}$

Antiderivative

$h=-\frac{1.67}{2}t^2+30t+C \Rightarrow t=0,\ h=0$

$0=0+0+C$

$C=0$

$\boxed{h=-\frac{1.67}{2}t^2+30t}$

At highest Point ... $V=0$

$0=-1.67t+30$

$-30=-1.67t$

$t=\frac{-30}{-1.67}=17.96$ seconds

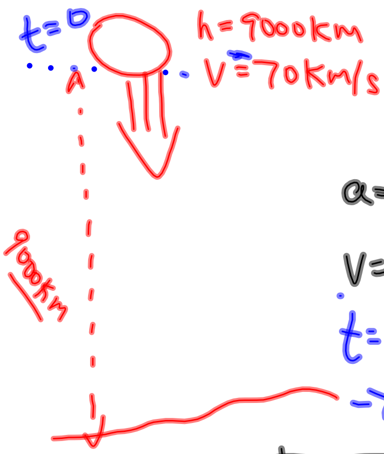
height @ $t=17.96$ sec

$h=-\frac{1.67}{2}(17.96)^2+30(17.96)$

$h=269.46$ m

Example:
A meteoroid falling to Earth is discovered when it is at an altitude of 9000 kilometers, traveling at a velocity of 70 kilometers per second. Assuming acceleration due to Earth's gravity is constant, and neglecting air resistance, how fast will the meteoroid be falling when it hits the ground? What will its acceleration be?

$71.249\,\frac{\text{km}}{\text{sec}}$



$$-9.8\,\frac{m}{s^2} \times \frac{1\ km}{10^3\ m} = -9.8\times10^{-3}\ km/s^2$$

$$a = -9.8\times10^{-3}$$

$$v = (-9.8\times10^{-3})t + C$$

$t=0,\ v=70\ km/s$

$$-70 = 0 + C$$
$$C = -70$$

$$\boxed{V = (-9.8\times10^{-3})t - 70}$$

$$h = -4.9\times10^{-3}t^2 - 70t + C$$

$t=0 \quad h = 9000km$

$$9000 = 0 - 0 + C$$
$$C = 9000$$

$$\boxed{h = -4.9\times10^{-3}t^2 - 70t + 9000}$$

When will $h=0$ . . .

$$t = \frac{70 \pm \sqrt{(70)^2 - 4(-0.0049)(9000)}}{2(-0.0049)}$$

$$t = \frac{70 \pm 71.249}{-0.0098} = -14\cancel{43}\ldots \text{ or } \underline{127.449\ sec}$$

Need velocity @ $t = \underline{127.449\ sec}$

$$V = -0.0098(127.449) - 70$$

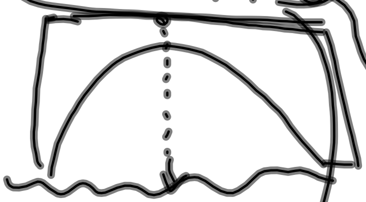
$$V = -71.249\ km/sec$$

Example:
You drop a rock off Quechee Gorge Bridge and it hits the water below about 3.2 seconds later. Approximately how high is the bridge?

50.176m

<u>Constant</u>: accel. due to gravity $= -9.8\,m/s^2$

$t=0, v=0, h=0$



$a=-9.8$

$v=-9.8t+c$

$\therefore c=0$

$v=-9.8t$

$h=-4.9t^2+c$

$c=0$

$h=-4.9t^2$

after 3.2 sec...

$h=-4.9(3.2)^2$

$h=-50.176m$

$\therefore$ height $= \underline{50.176m}$

Homework

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