

10. Sketch the graph of the function  
 $y = 2(x - 1)^2 - 8$  using transformations.

Then, copy and complete the table.

Vertex	$(1, -8)$
Axis of Symmetry	$x = 1$
Direction of Opening	Up
Domain	$x \in \mathbb{R}$
Range	$y \geq -8$
x-Intercepts	$(3, 0) \quad (-1, 0)$
y-Intercept	$(0, -6)$

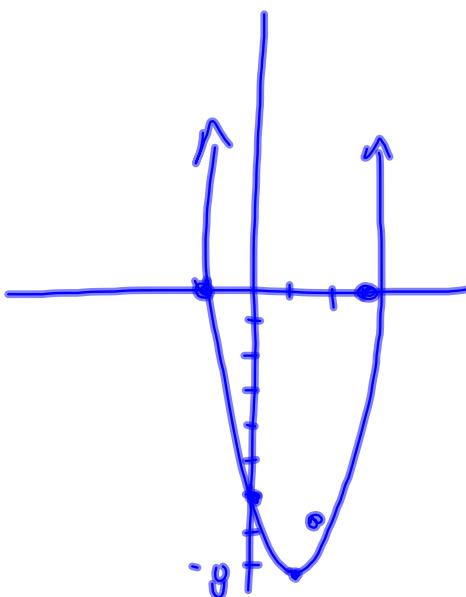
$$0 = 2(x-1)^2 - 8$$

$$8 = 2(x-1)^2$$

$$\sqrt{4} = \sqrt{(x-1)^2}$$

$$\pm 2 = x - 1$$

$$\begin{cases} \pm 2 = x \\ x = 3 \text{ or } x = -1 \end{cases}$$



$$(ii) y = -3x^2 - 18x - 6$$

$$y = -3(x+6x+9) - 6 + 27$$

$$y = -3(x+3)^2 + 21$$

$$0 = -3(x+3)^2 + 21$$

$$\frac{-21}{-3} = \frac{-3(x+3)^2}{-3}$$

$$\sqrt{7} = \sqrt{(x+3)^2}$$

$$\pm\sqrt{7} = x+3$$

$$x = -3 \pm \sqrt{7}$$

$$0 = -3x^2 - 18x - 6$$

$$0 = 3x^2 + 18x + 6$$

$$0 = x^2 + 6x + 2$$

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(2)}}{2}$$

$$x = \frac{-6 \pm \sqrt{28}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{7}}{2}$$

$$x = -3 \pm \sqrt{7}$$

Vertex	$(-3, 21)$
Axis of Symmetry	$x = -3$
Direction of Opening	Down
Domain	$x \in \mathbb{R}$
Range	$y \leq 21$
x-Intercepts	$x = -3 \pm \sqrt{7}$
y-Intercept	$(0, -6)$

Austin and Yuri were asked to convert the function  $y = -6x^2 + 72x - 20$  to vertex form. Their solutions are shown.

*Austin's solution:*

$$\begin{aligned}y &= -6x^2 + 72x - 20 \\y &= -6(x^2 + 12x) - 20 \\y &= -6(x^2 + 12x + 36 - 36) - 20 \\y &= -6[(x^2 + 12x + 36) - 36] - 20 \\y &= -6[(x + 6)^2 - 36] - 20 \\y &= -6(x + 6)^2 + 216 - 20 \\y &= -6(x + 6)^2 + 196\end{aligned}$$

*Yuri's solution:*

$$\begin{aligned}y &= -6x^2 + 72x - 20 \\y &= -6(x^2 - 12x) - 20 \\y &= -6(x^2 - 12x + 36 - 36) - 20 \\y &= -6[(x^2 - 12x + 36) - 36] - 20 \\y &= -6[(x - 6)^2 - 36] - 20 \\y &= -6(x - 6)^2 - 216 - 20 \\y &= -6(x - 6)^2 + 236\end{aligned}$$

Kihun's

$$-\frac{y}{6} = x^2 - 12x + \frac{10}{3}$$

$$-\frac{y}{6} = (x^2 - 12x + 36) + \frac{10}{3} - \frac{36}{1}$$

$$-\frac{y}{6} = (x - 6)^2 - \frac{108}{3} (6)^2$$

$$y = 6(x - 6)^2 + 216$$

Martine's teacher asks her to complete the square for the function

$y = -4x^2 + 24x + 5$ . After looking at her solution, the teacher says that she made four errors in her work. Identify, explain, and correct her errors.

*Martine's solution:*

$$y = -4x^2 + 24x + 5$$

$$y = -4(x^2 + 6x) + 5$$

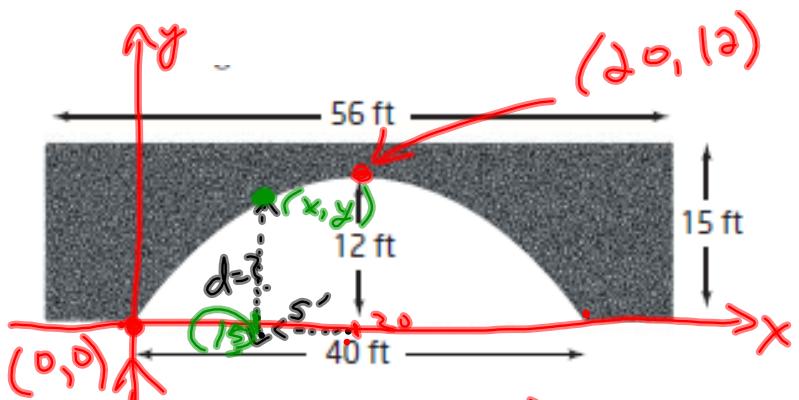
$$y = -4(x^2 + 6x + 36) - 36 + 5$$

$$y = -4[(x^2 + 6x + 36) - 36] + 5$$

$$y = -4[(x + 6)^2 - 36] + 5$$

$$y = -4(x + 6)^2 - 216 + 5$$

$$y = -4(x + 6)^2 - 211$$



$$y = a(x-h)^2 + k$$

$$y = a(x-20)^2 + 12$$

$$0 = a(0-20)^2 + 12$$

$$\frac{-12}{400} = \frac{400a}{700}$$

$$-\frac{3}{100} = a$$

$$y = -\frac{3}{100}(x-20)^2 + 12$$

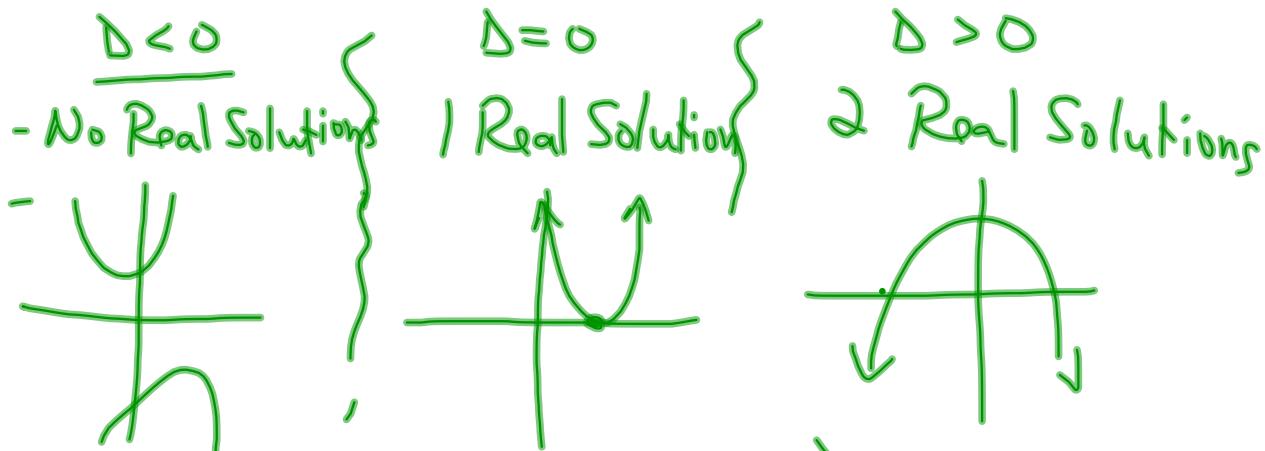
$$y = -\frac{3}{100}(15-20)^2 + 12$$

$$\underline{y = 11.25 \text{ feet}}$$

Nature of roots:

$$\text{Discriminant: } D = b^2 - 4ac$$

Determine the value of  $k$  so that  $y = -5x^2 + 3kx - 2k$  will only have ONE real root.



$$-5x^2 + 3kx - 2k = 0$$

$$\underline{D = 0}$$

$$b^2 - 4ac = 0$$

$$(3k)^2 - 4(-5)(-2k) = 0$$

$$9k^2 - 40k = 0$$

$$k(9k - 40) = 0$$

$$k = 0 \text{ or } 9k - 40 = 0$$

$$\frac{9k}{9} = \frac{40}{9}$$

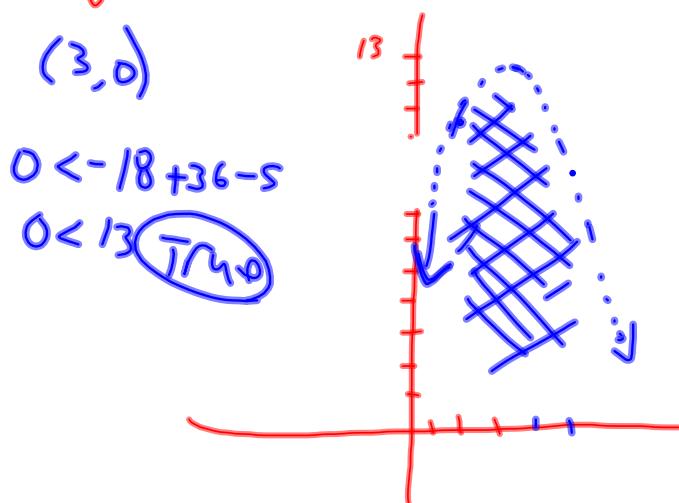
$$k = \frac{40}{9}$$

$D$  is a perfect square  
Rational Roots

Sketch:  $y < -2x^2 + 12x - 5$

$$y = -2(x^2 - 6x + 9) - 5 + 18$$

$$y = -2(x-3)^2 + 13$$



Write as piecewise:

$$(1) f(x) = |x-7|$$

BBP

$$\begin{array}{l} x-7 > 0 \\ x > 7 \end{array} \Rightarrow f(x) = x-7$$

BBN

$$\begin{array}{l} x-7 \leq 0 \\ x \leq 7 \end{array} \Rightarrow f(x) = -x+7$$

$$f(x) = \begin{cases} x-7 & ; f x > 7 \\ -x+7 & ; f x \leq 7 \end{cases}$$

$$(2) f(x) = |x^2-x-6| + 3x$$

BBP:  $x < -2$  or  $x > 3$

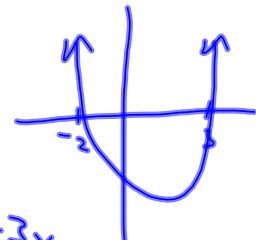
$$x^2-x-6 > 0$$

$$(x-3)(x+2) > 0$$

$$x = 3, -2$$

$$f(x) = x^2 - x - 6 + 3x$$

$$f(x) = x^2 + 2x - 6$$



BBN:  $-2 \leq x \leq 3$

$$\begin{aligned} f(x) &= -x^2 + x + 6 + 3x \\ f(x) &= -x^2 + 4x + 6 \end{aligned}$$

$$f(x) = \begin{cases} -x^2 + 4x + 6 & ; f -2 \leq x \leq 3 \\ x^2 + 2x - 6 & ; f x < -2 \text{ or } x > 3 \end{cases}$$

## Absolute Value Equation

Solve:

$$|3x - 5| = 2x + 7$$

BBP

$$3x - 5 \geq 0$$

$$x \geq \frac{5}{3}$$

$$3x - 5 = 2x + 7$$

$$(x = 12)$$

OK!

BBN:

$$3x - 5 < 0$$

$$x < \frac{5}{3}$$

$$-3x + 5 = 2x + 7$$

$$-5x = 2$$

$$(x = -\frac{2}{5})$$

OK!

$$|3x^2 - 13x - 10| = x^2 + 2 - x$$

BBP:

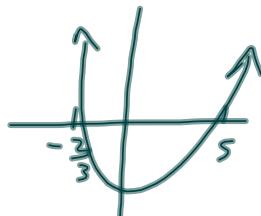
$$3x^2 - 13x - 10 > 0$$

$$3x^2 - 15x + 2x - 10 = 0$$

$$3x(x-5) + 2(x-5) = 0$$

$$(x-5)(3x+2) = 0$$

$$x = 5 \text{ or } -\frac{2}{3}$$



$$x < -\frac{2}{3} \text{ or } x > 5$$

$$3x^2 - 13x - 10 = x^2 - x + 2$$

$$2x^2 - 12x - 12 = 0$$

$$x^2 - 6x - 6 = 0 \Rightarrow x = \frac{6 \pm \sqrt{36 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{6 \pm \cancel{\sqrt{60}}}{2} = 2\sqrt{15}$$

$$x = 3 \pm \sqrt{15}$$

BBN:

$$-\frac{2}{3} \leq x \leq 5$$

$$-3x^2 + 13x + 10 = x^2 - x + 2$$

$$-4x^2 + 14x + 8 = 0$$

$$2x^2 - 7x - 4 = 0$$

$$2x^2 - 8x + x - 4 = 0$$

$$2x(x-4) + (x-4) = 0$$

$$(x-4)(2x+1) = 0$$

$$x = 4 \text{ or } x = -\frac{1}{2}$$

$$x = 3 + \sqrt{15}$$

$$x = 6.87$$

$$x = 3 - \sqrt{15}$$

$$x = -0.87$$

$$|x - s| \leq 2x + 6$$

BBP

$x - s \geq 0$  then  $x - s \leq 2x + 6$

$x \geq s$        $-x \leq 1$        $x \geq -\frac{1}{3}$

$x \geq s$

Both must be true!!

BBN

$x - s < 0$  then  $-x + s \leq 2x + 6$

$x < s$        $-3x \leq 1$        $x \geq -\frac{1}{3}$

$-\frac{1}{3} \leq x < s$

Both ...

Final Solution.... Join the 2 cases...

