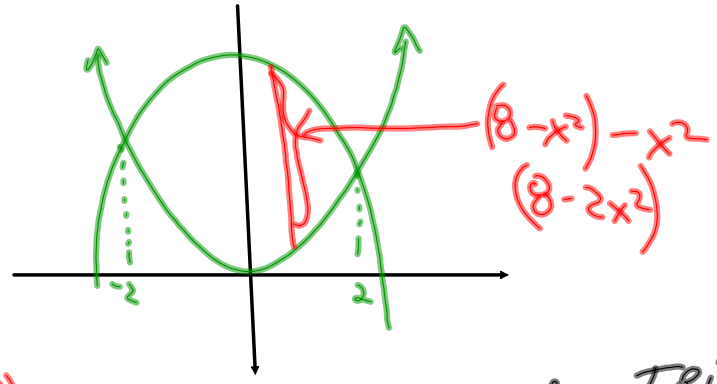
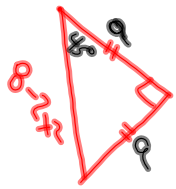
The base of the volume is the region bounded by the curves $y = 8 - x^2$ and $y = x^2$.
The cross sections perpendicular to the x-axis are:

a. Squares
b. Equilateral triangles
c. Isosceles right triangles with leg on the base
d. Isosceles right triangles with hypotenuse on the base
e. Semi-circles
f. Quarter-circles
g. Isosceles trapezoids with 60° base angles



d)



Pythagoras

$a^2+a^2=(8-2x^2)^2$

$2a^2=(8-2x^2)^2$

$a^2=\frac{(8-2x^2)^2}{2}$

$a=\frac{8-2x^2}{\sqrt{2}}$

TRig

$\sin 45^\circ = \frac{a}{8-2x^2}$

$\frac{1}{\sqrt{2}} = \frac{a}{8-2x^2}$

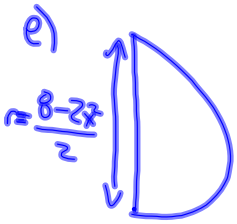
$a=\frac{8-2x^2}{\sqrt{2}}$

$A=\frac{1}{2}\left(\frac{8-2x^2}{\sqrt{2}}\right)^2$

$A=\frac{(8-2x^2)^2}{4}$

$\frac{1}{4}\int_{-2}^{2}(8-2x^2)^2\,dx$

e)



$A=\frac{1}{2}\pi r^2$

$A=\frac{1}{2}\pi(4-x^2)^2$

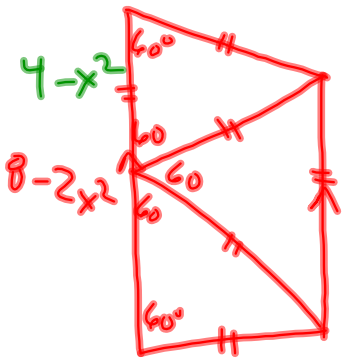
$V=\frac{1}{2}\pi\int_{-2}^{2}(4-x^2)^2\,dx$

f)

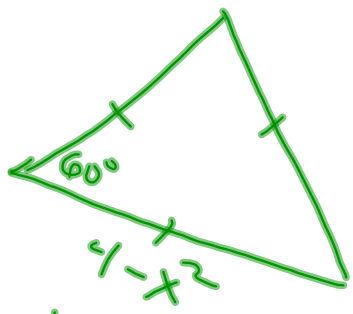
$A=\frac{1}{4}\pi r^2$

$A=\frac{1}{4}\pi(8-2x^2)^2$

$\frac{\pi}{4}\int_{-2}^{2}(8-2x^2)^2\,dx$



$$\cancel{A = \frac{1}{2}h(a+b)}$$



$$A = \frac{1}{2}(4-x^2)^2 \sin 60°$$

$$A = \frac{\sqrt{3}}{4}(4-x^2)^2 \times 3$$

$$A = \frac{3\sqrt{3}}{4}(4-x^2)^2$$

$$\frac{3\sqrt{3}}{4}\int_{-2}^{2}(4-x^2)^2\,dx$$

Warm Up...

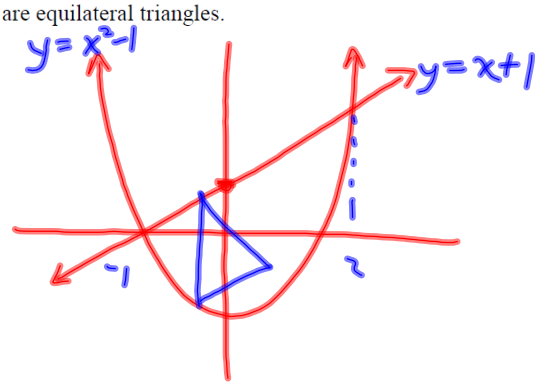
7. Find the volume of the solid whose base is bounded by the graphs of $y = x+1$ and $y = x^2 - 1$ whose cross sections perpendicular to the x-axis are equilateral triangles.

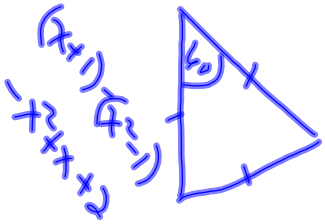
$x+1 = x^2-1$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,-1$





$A = \frac{1}{2}(-x^2+x+2)^2 \sin 60°$

$$V = \frac{\sqrt{3}}{4}\int_{-1}^{2}(-x^2+x+2)^2\,dx$$

$$\frac{\sqrt{3}}{4}\int_{-1}^{2}(x^4-2x^3-3x^2+4x+4)\,dx$$

$(-x^2+x+2)(-x^2+x+2)$

$x^4-x^3-2x^2-x^3+x^2+2x-2x^2+2x+4$

$x^4-2x^3-3x^2+4x+4$

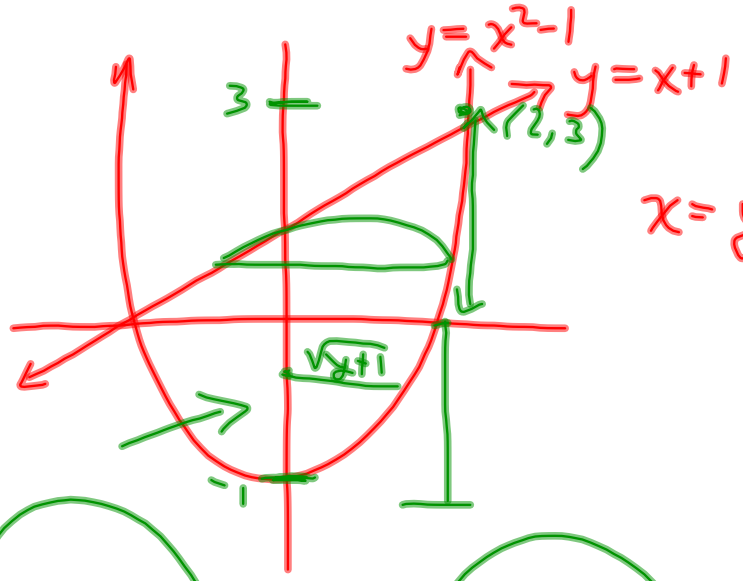
$$\frac{\sqrt{3}}{4}\left(\frac{x^5}{5}-\frac{1}{2}x^4-x^3+2x^2+4x\right)\Big|_{-1}^{2}$$

$$= \frac{\sqrt{3}}{4}\left[\left(\frac{32}{5}-\cancel{8-8+8+8}\right)-\left(\frac{-1}{5}-\frac{1}{2}+1+2-4\right)\right]$$

$$= \frac{\sqrt{3}}{4}\left(\frac{32}{5}+\frac{1}{5}+\frac{1}{2}+1\right)$$

$$= \frac{\sqrt{3}}{4}\left(\frac{64+2+5+10}{10}\right)$$

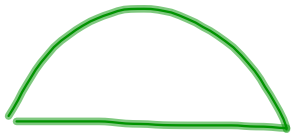
$$= \frac{81\sqrt{3}}{40}u^3$$



$x=y-1$

$y+1=x^2$

$\pm\sqrt{y+1}=x$



$A=\frac{1}{2}\pi\left(\sqrt{y+1}\right)^2$

$\sqrt{y+1}-(y-1)$

$r=\dfrac{\sqrt{y+1}-y+1}{2}$

$A=\frac{1}{2}\pi\left(\dfrac{\sqrt{y+1}-y+1}{2}\right)^2$

$A=\frac{1}{8}\pi\left(\sqrt{y+1}-y+1\right)^2$

$$V=\frac{\pi}{2}\int_{-1}^{0}(y+1)\,dy+\frac{\pi}{8}\int_{0}^{3}\left(\sqrt{y+1}-y+1\right)^2dy$$

Practice Questions...

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#3, 5, 7, 9, 11, 13, 33

Midterm: Wednesday, March 20 !!