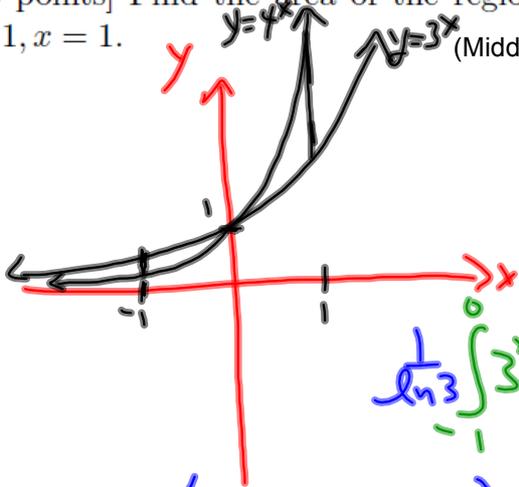


Midterm Review...

[5 points] Find the area of the region bounded by the curves $y = 3^x$, $y = 4^x$, $x = -1$, $x = 1$.



(Middlebury College)

$$\int_{-1}^0 (3^x - 4^x) dx + \int_0^1 (4^x - 3^x) dx$$

$$\frac{1}{\ln 3} \int_{-1}^0 3^x dx - \frac{1}{\ln 4} \int_{-1}^0 4^x dx + \frac{1}{\ln 4} \int_0^1 4^x dx - \frac{1}{\ln 3} \int_0^1 3^x dx$$

$$= \left(\frac{1}{\ln 3} 3^x - \frac{1}{\ln 4} 4^x \right) \Big|_{-1}^0 + \left(\frac{1}{\ln 4} 4^x - \frac{1}{\ln 3} 3^x \right) \Big|_0^1$$

$$= \left[\left(\frac{1}{\ln 3} - \frac{1}{\ln 4} \right) - \left(\frac{1}{3\ln 3} - \frac{1}{4\ln 4} \right) \right] + \left[\left(\frac{4}{\ln 4} - \frac{3}{\ln 3} \right) - \left(\frac{1}{\ln 4} - \frac{1}{\ln 3} \right) \right]$$

$$= -\frac{1}{\ln 3} + \frac{2}{\ln 4} - \frac{1}{3\ln 3} + \frac{1}{4\ln 4}$$

$$= -\frac{4}{3\ln 3} + \frac{9}{4\ln 4}$$

$$\text{Find } \lim_{x \rightarrow -2} \frac{x+2}{\ln(x+3)} = \frac{0}{0} \quad \text{L'Hospital's Rule} \quad = 1$$

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{1}{\left(\frac{1}{x+3}\right)'(1)} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

$$\text{Find } \lim_{x \rightarrow \infty} \frac{3^x}{x^2 + x - 1} = \frac{\infty}{\infty}$$

∞ (D.N.E.)

$$\lim_{x \rightarrow \infty} \frac{3^x \ln 3}{2x + 1}$$

$$\lim_{x \rightarrow \infty} \frac{3^x \ln^2 3}{2} = \frac{\infty}{2}$$

$\rightarrow \infty$
 \therefore D.N.E.



$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = \frac{\infty}{\infty}$$

$= 0$

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2} x^{-1/2}}$$

$$\Rightarrow \frac{1}{x} \div \frac{1}{2\sqrt{x}}$$

$$\frac{1}{x} \cdot \frac{2\sqrt{x}}{1} = \frac{2\sqrt{x}}{x}$$

$$\lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{x}$$

$$\lim_{x \rightarrow \infty} \frac{x^{-1/2}}{1}$$

$$\lim_{x \rightarrow \infty} \frac{\left(\frac{1}{\sqrt{x}}\right)}{1} = \frac{0}{1}$$

$\frac{1}{\infty} = 0$

Find $\lim_{x \rightarrow 0} \frac{5x - \tan(5x)}{x^3}$

$= -\frac{125}{3}$

$\lim_{x \rightarrow 0} \frac{5 - \sec^2 5x (5)}{3x^2} \quad - 5(\sec 5x)^2$

$\lim_{x \rightarrow 0} \frac{-10(\sec 5x)(\sec 5x \tan 5x)(5)}{6x}$

$\lim_{x \rightarrow 0} \frac{-50 \sec^2 5x \tan 5x}{6x}$

$\sec 0 = 1$
 $\tan 0 = 0$

$\lim_{x \rightarrow 0} \frac{-50 [2(\sec 5x)'(\sec 5x \tan 5x)(5) \tan 5x + \sec^2 5x (\sec^2 5x)(5)]}{6}$

$= \frac{-50(0 + 5)}{6}$
 $= \frac{-250}{6} = \frac{-125}{3}$

Let $f(x) = x^2 - 5x + 1$ on $[0, 2]$. Find c from the mean value theorem, i.e. c in $[0, 2]$ and such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(0) = 1$$

$$f(2) = -5$$

$$f(x) = (x^2 - 5x + \frac{25}{4}) + 1 - \frac{25}{4}$$

$$f(x) = (x - \frac{5}{2})^2 - \frac{21}{4}$$

$$f'(x) = 2x - 5$$

$$2x - 5 = -3$$

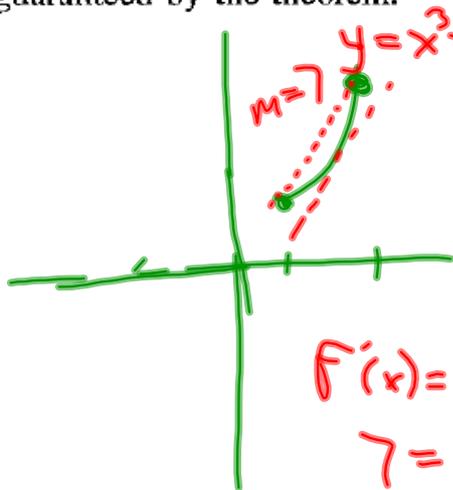
$$2x = 2$$

$$x = 1$$

$$f'(c) = -3$$

$$M = \frac{-5 - 1}{2 - 0} = -3$$

Example 3 Let $f(x) = x^3 + 1$. Show that f satisfies the hypotheses of the Mean-Value Theorem on the interval $[1, 2]$ and find all values of c in this interval whose existence is guaranteed by the theorem.



$$f(1) = 2 \quad (1, 2)$$

$$f(2) = 9 \quad (2, 9)$$

$$M = \frac{9 - 2}{2 - 1} = 7$$

$$f'(x) = 3x^2$$

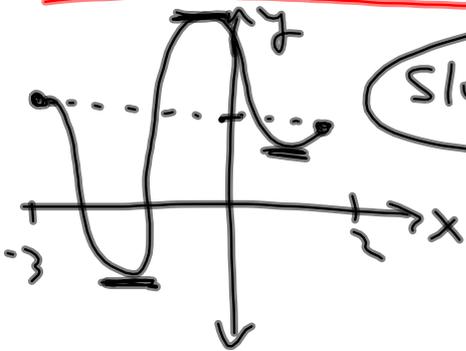
$$7 = 3x^2$$

$$x^2 = \frac{7}{3}$$

$$x = \pm \sqrt{\frac{7}{3}}$$

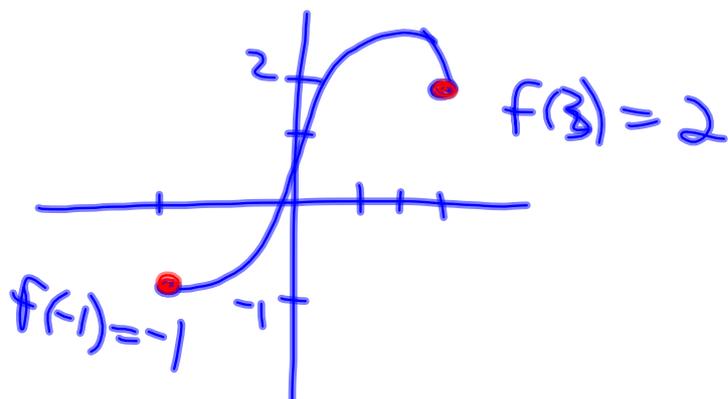
$$\therefore x = +\sqrt{\frac{7}{3}}$$

Rolle's Theorem (Special case of MVT)



Slope of secant = 0

Intermediate Value Theorem

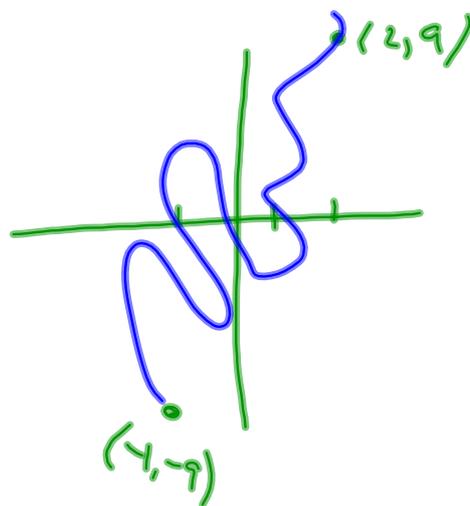


Prove that $f(x) = 2x^3 - 7$ has
at least one zero on the interval
 $[-1, 2]$

$$f(-1) = 2(-1)^3 - 7 = -9$$

$$f(2) = 2(2)^3 - 7 = 9$$

\therefore By IVT there
must be at least
one x , such that $f(x) = 0$



Attachments

Worksheet - Sketching Angles in Radians.doc