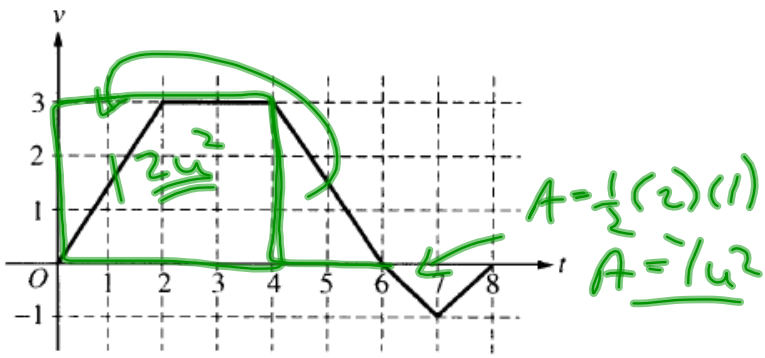
Warm Up

Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$



A bug begins to crawl up a vertical wire at time $t=0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

What is the total distance the bug traveled from $t=0$ to $t=8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

$f'(x) = 0$

$$\frac{d}{dx}\int_{-2}^{x^2-3x} e^{t^2}\,dt = 0$$

$$\cancel{e^{(x^2-3x)^2}}\,(2x-3) = 0$$

$2x - 3 = 0$

$x = \frac{3}{2}$

$e^{\text{Blah! Blah!}} \neq 0$

If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

Integral Property: $\int [f(x)+g(x)]\,dx$

$= \int f(x)\,dx + \int g(x)\,dx$

$\int_a^b (f(x)+5)\,dx$

$= \int_a^b f(x)\,dx + \int_a^b 5\,dx$

$= (a+2b) + 5x \Big|_a^b$

$= a+2b+(5b-5a)$

$= 7b-4a$

Techniques of Integration

I. Substitution Technique

Examples:

$$\int 3x\left(x^2+1\right)^{20} dx$$

$u^n \cdot du$

$$\frac{3}{2}\int (x^2+1)^{20}\, 2x\,dx$$

$$= \frac{1}{14}(x^2+1)^{21} + C$$

u-substitution

let $u = x^2+1$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\frac{du}{2} = x\,dx$$

Substitute . . .

$$\frac{3}{2}\int u^{20}\,du$$

$$\frac{3}{2}\left(\frac{u^{21}}{21}\right) + C$$

$$= \frac{1}{14}u^{21} + C$$

Re-Substitute . . .

$$= \frac{1}{14}(x^2+1)^{21} + C$$

$$\int \sin^2 x \cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$\cancel{u = \cos x}$

$\cancel{du = -\sin x\,dx}$

$$\int u^2\,du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{\sin^3 x}{3} + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx$

$2du = \frac{dx}{\sqrt{x}}$

$= \int 2e^u du$

$= 2\int e^u du$

$= 2e^u + C$

$= 2e^{\sqrt{x}} + C$

$\int x^2\sqrt{x-1}\,dx$ ← ???

This one is a little different

$u = x-1 \Longrightarrow u+1 = x$

$\frac{du}{dx} = 1$ $\qquad x^2 = (u+1)^2$

$du = dx$

$$\int (u+1)^2 u^{1/2}\,du$$

$$\int (u^2+2u+1)u^{1/2}\,du$$

$$\int \left(u^{5/2}+2u^{3/2}+u^{1/2}\right)du$$

$$= \frac{2}{7}u^{7/2} + \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

$$\int_0^2 x^2\sqrt{x^3+1}\,dx$$ ← What about a definite integral???

Look at both methods....

$u = x^3 + 1$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\underline{x=2}$

$u = 2^3 + 1$

$u = 9$

$\underline{x=0}$

$u = 0^3 + 1$

$u = 1$

$$\frac{1}{3}\int_1^9 u^{1/2}\,du$$

$$= \frac{2}{9}u^{3/2}\Big|_1^9$$

$$= \frac{2}{9}\left[(9)^{3/2} - (1)^{3/2}\right]$$

$$= \frac{2}{9}(27 - 1)$$

$$= \frac{52}{9}$$