Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on…

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

$\cos^2\theta = (\cos\theta)^2$

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

$3x^7 - 2x^{10} + 8x^3 \leftarrow$ Degree 10

$3x^{10} + 2y^{\overset{15}{2+13}} \leftarrow$ Degree 15

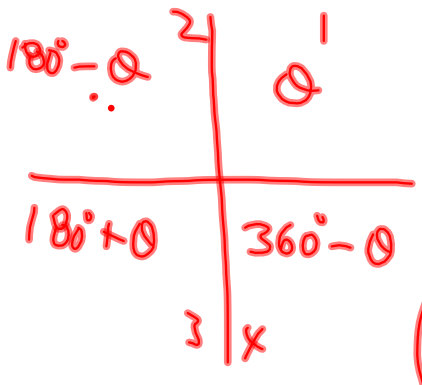
Let's start with basic LINEAR trigonometric equations...

Solve: $\sin\theta = 0.9659\ ,\ -360° < x < 720°$

...Pre-Calculus 110

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.



(Ref ∡ 75°, Q 1,2)

$0° < \theta < 90°$

$360° \rightarrow 720°$

$\rightarrow \pm 360°$

$\theta = 75°, 105°, 435°, 465°$

$-360° < \theta < 0°$

$\rightarrow -360°$

$\theta = -285°, -255°$

cos (−) −360° 360°

Solve: $\sec\theta = -1.3054$, $-2\pi \le x \le 2\pi$ Radians

Always (+) for Reference Angle

Option 1: Do in degrees then convert

(Ref∡: 40° , Q 2,3)

$\cos^{-1}(1.3054^{-1})$

$\boxed{x^{-1}}$

$\boxed{\frac{1}{x}}$

$\theta = 140°, 220°, -220°, -140°$

−360° −360°

$\pm\frac{140\pi}{180}, \pm\frac{220\pi}{180}$

$\pm\frac{7\pi}{9}, \pm\frac{11\pi}{9}$

Switch to Radians

$\sec\theta = -1.3054\ ,\ -2\pi \le x \le 2\pi$

(Ref∡ 0.698, Q 2,3)

$\theta = 2.444,\ 3.840$

$\downarrow -2\pi \qquad \downarrow -2\pi$

$\theta = -3.840,\ -2.444$

$\pi - \theta$	θ
$\pi + \theta$	$2\pi - \theta$

$\theta = \pm 2.444,\ \pm 3.840$

Ex. $\sqrt{2}\cos\theta + 1 = 0, -360° \le \theta \le 720°$

$\sqrt{2}\cos\theta = -1$

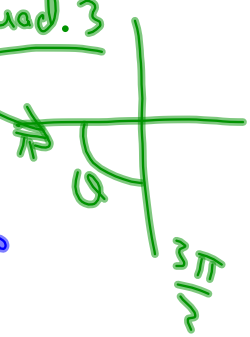
$\cos\theta = \frac{-1}{\sqrt{2}}$

$\pi \le \theta \le \frac{3\pi}{2}$

Quad. 3

(Ref ∡: 45°, Q 2, 3)

$\theta = \pm 135°, \pm 225°, 495°, 585°$



$2w + 7 = 5$

$2w = -2$

$w = -1$

Ex. $\sin x + 1 = 0, \; -2\pi \le x \le 4\pi$

$\sin x = -1$ $\Rightarrow$ If trig. Ratio $= \pm 1$ OR 0

$\Rightarrow$ Quadrantal Angle !!

$\Rightarrow$ Use Unit Circle

$x = \frac{3\pi}{2}, \frac{7\pi}{2}, -\frac{\pi}{2}$

Your Turn

Solve each trigonometric equation in the specified domain.

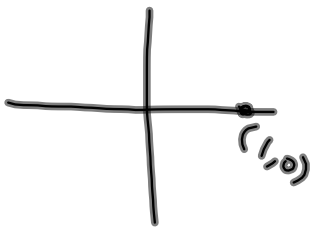
a) $3\cos\theta - 1 = \cos\theta + 1,\ -2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0,\ 0° \le x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$

$2\cos\theta = 2$

$\cos\theta = 1$



$\theta = 0, 2\pi, -2\pi$

b) $4\sec x = -8$

$\sec x = -2$

(Ref ∠ 60°, Q 2, 3)

$x = 120°, 240°$

Practice Questions

Pg. 202
#10, 11, 12

$x \to 1$

$\cos x = 1$