Solving Logarithmic Equations

STEPS... (1) Write left side & right side as a single logarithm

NOTE: $\log_a a = 1$

$2^3 = 2^{x+7}$

(2) Set arguments equal & solve the equation

✱ (3) Check for extraneous roots

$\log_w 7 = \log_w (x+3)$

EXAMPLE #1: $\log_3 x - \log_3 4 = \log_3 12$

$\log_3 \left(\frac{x}{4}\right) = \log_3 12$

Argument = Argument

$\frac{x}{4} = 12$

$x = 48$

check for extraneous Root

EXAMPLE #2:

Solve the following equation... $\log_{10}(x+2) + \log_{10}(x-1) = 1$

$\log_{10}[(x+2)(x-1)] = 1$

$\log_{10}(x^2+x-2) = 1$

$10^1 = x^2+x-2$

$0 = x^2+x-12$

$0 = (x+4)(x-3)$

$x \neq -4, 3$

$\log_b b^n = n$

$\log_{10}(x^2+x-2) = \log_{10} 10$

$\therefore x^2+x-2 = 10$

EXAMPLE #3:

Solve: $\log_2(x+1) + \log_2(x-1) = 3$

$\log_2 (x+1)(x-1) = 3$

$2^3 = x^2 - 1$

$0 = x^2 - 9$

$0 = (x-3)(x+3)$

$x = 3$ OR $x \neq -3$ extraneous

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$$\frac{\log_3\left(\frac{x}{4}\right)}{\log_3} = \frac{\log_3 12}{\log_3}$$

$$\frac{x}{4} = 12$$

$$x = 48$$

$$\frac{\sin 60}{\sin} = \frac{x}{\sin}$$

Exponential Equations

What if both sides can not be written to powers of a common base?

Example: $3^x = 30$

What would this equation be if expressed as a logarithmic statement?

$\log_3 30 = x$ Can this be determined using a calculator?

Here is a new method to solve exponential equations...

- Particularly effective when unable to express both sides as a power of a common base

Key property of equations...

- As long as you perform the same operation to BOTH sides of an equation, equality will be maintained

$3^x = 30$

Take the common logarithm of both sides...or natural logarithm

Why base 10 or base "e" ?

$\log 3^x = \log 30$

$x \log 3 = \log 30$

$\frac{x \log 3}{\log 3} = \frac{\log 30}{\log 3}$

$x = 3.096$

$2^x = 16$

$\log 2^x = \log 16$

$x \log 2 = \log 16$

$x = \frac{\log 16}{\log 2} = 4$

Example: $6^{2x-3} = 8^{x+1}$

$\log 6^{2x-3} = \log 8^{x+1}$

$(2x-3)\log 6 = (x+1)\log 8$

$(2x-3)(5)$

$10x - 15$

$(2\log 6)x - 3\log 6 = x\log 8 + \log 8$

$(2\log 6)x - (\log 8)x = \log 8 + 3\log 6$

$$\frac{x(2\log 6 - \log 8)}{2\log 6 - \log 8} = \frac{(\log 8 + 3\log 6)}{(2\log 6 - \log 8)}$$

$x = 4.956$

$$\log_M M^x = x$$

$$\log_2(x+3) = 5$$

$$\Downarrow$$

$$2^5 = x+3$$

$$\frac{\log_3(x^2+4)}{\log_3 3} = \frac{\log_3 10}{\log_3 3}$$