$268\ 435\ 456$

$2^{?} = 268\ 435\ 456$

2^{28}

$129\ 140\ 163$

$3^{?} = 129\ 140\ 163$

3^{17}

$5^{?} = 10\ 000$

$\underline{5.723}$

$3^{x} = 129\ 140\ 163$

$x = \dfrac{\log 129\ 140\ 163}{\log 3}$

$\log 2^{x} = \log 268\ 435\ 456$

$x = \dfrac{\log 268\ 435\ 456}{\log 2}$

$\log_a x$

$\log_3 7 = 1.771$

$\frac{\log 7}{\log 3}$

$\log b^x = \log N$

$x \log b = \log N$

$$x = \frac{\log N}{\log b}$$

This process leads to a very useful formula...

Have you ever thought...

"b raised to what power will give me some number N?"

ie. $\log_b N = x$ or $b^x = N$

Solve the equation: $b^x = N$

$$x\log_{10} b = \log_{10} N$$

$$x = \frac{\log_{10} N}{\log_{10} b}$$

Could we have taken the logarithm of each side to any base we had chosen?

This leads to the change of base formula:

$b^{?} = N$

$$\log_b N = \frac{\log_a N}{\log_a b}$$

Evaluate each of the following:

$$\log_5 45 = \frac{\log 45}{\log 5} = 2.3652\ldots$$

$$\log_3 7 = 1.771$$

"e"

$$\frac{\ln 45}{\ln 5} = 2.3652\ldots$$

$$\frac{\log_{13} 45}{\log_{13} 5} = 2.3652$$

$2^{x+5} = 4096$

$= 2^{12}$

$\log_2 4096$

$$2^{?} = 4096$$

$$\log_2 4096$$

$$\frac{\log 4096}{\log 2}$$

$$= 12$$

8.e) $\log_2 \sqrt{x^2+4x} = \frac{5}{2}$

$$\left(2^{5/2}\right)^2 = \left(\sqrt{x^2+4x}\right)^2$$

$$2^5 = x^2+4x$$

$$0 = x^2+4x-32$$

$$0 = (x+8)(x-4)$$

$\underline{\underline{x=-8}}$ or $\underline{\underline{x=4}}$

$$x^2+6x=0$$

$$x(x+6)=0$$

$x=0$ $\quad$ $\cancel{x=-6}$

$2^3 = x(x+2)$ $\quad$ or $\quad$ $\log_2(x(x+2)) = 3$

$$\log_2(x(x+2)) = \log_2 \underline{\underline{2}}^3$$

$$x(x+2) = 2^3$$

Warm-up...

Solve the following... $\log\left(\dfrac{3^x \bullet 2^{x+1}}{4^{3x}}\right) = \log\left(5^{2x}\right)$

0.124

$x\log 3 + (x+1)\log 2 - 3x\log 4 = 2x\log 5$

$x(\log 3 + \log 2 - 3\log 4 - 2\log 5) = -\log 2$

$x = \dfrac{0.124}{\cancel{(\log_5 x)}}$

Solve the following... $\dfrac{2\log_5 2 + \log_5(x+8)}{\cancel{\log_5 x}} = 2\log_5 x$

$\log_5 4 + \log_5(x+8) = \log_5 x^2$

$\log_5(4(x+8)) = \log_5 x^2$

$\therefore 4x + 32 = x^2$

$x^2 - 4x - 32 = 0$

$(x-8)(x+4) = 0$

$x = 8$, $\cancel{-4}$ extraneous

HOMEWORK...

Pg. 413-415

9-18

EXTRA ! EXTRA!

Solve: $$\frac{2^x \bullet 5^{3x-1}}{4^{2x+1}} = \frac{5\left(7^{5x}\right)}{3^{2x+3}}$$

a) $\log_6\left(\frac{2x+1}{x-1}\right) = \log_6 5$

$\cancel{(x-1)}\,\frac{2x+1}{\cancel{x-1}} = 5(x-1)$

$2x+1 = 5x-5$

$-3x = -6$

$x = 2$

b)

$\log_2(x^2+2x) = \log_2 2^3$

c) $\log_2(x(x+2)) = 3$

$2^3 = x^2+2x$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x = \cancel{-4}, 2$

Pg. 413 - 415

#9-18

#20, 21, 22,

C4

$10E-4 \Rightarrow 10 \times 10^{-4} = 10^{-3}$

1×10

$$3^{x+2} = \frac{27^x}{9^{3x-4}}$$

$$3^{x+2} = \frac{15^x}{8^{2x+3}}$$

1. Solve: $\frac{2 \cdot 6^{x+1}}{2} = \frac{18}{2}$.

(A) $\frac{\log 9 - 1}{\log 6}$

(B) $\frac{\log 9}{\log 6} - 1$

(C) $\frac{\log 18}{\log 12} - 1$

(D) $\frac{\log 18 - 1}{\log 12}$

$6^{x+1} = 9$

$\frac{(x+1)\cancel{\log 6}}{\cancel{\log 6}} = \frac{\log 9}{\log 6}$

$x = \frac{\log 9}{\log 6} - 1$

$\log(2 \cdot 6^{x+1}) = \log 18$

$\log 2 + (x+1)\log 6 = \log 18$

$\log 2 + x\log 6 + \log 6 = \log 18$

$x\log 6 = \log 18 - \log 6 - \log 2$

$x = \frac{\log 18 - \log 6 - \log 2}{\log 6}$

$x = \frac{\log 18 - \log 2}{\log 6} - \frac{\log 6}{\log 6}$

$x = \frac{\log\left(\frac{18}{2}\right)}{\log 6} - 1$

$x = \frac{\log 9}{\log 6} - 1$

2. Which is equivalent to $3\left[\frac{1}{2}\log_4 A + \log_4 B - \log_4 C\right]$?

$3\log_4\left(\frac{A^{1/2}B}{C}\right)$

$\log_4\left(\frac{\sqrt{A}B}{C}\right)^3$

(A) $\log_4 \frac{\left(B\sqrt{A}\right)^3}{C}$

(B) $\frac{3}{2}\log_4 \frac{AB}{C}$

(C) $\log_4\left(\sqrt{\frac{AB}{C}}\right)^3$

(D) $\log_4\left(\frac{B\sqrt{A}}{C}\right)^3$

3. Algebraically solve: $\log_2(x+2)+\frac{1}{3}\log_2(8x^3)=16^{\frac{1}{2}}$.

$$\frac{1}{3}\log_2 8x^3$$
$$\log_2(8x^3)^{1/3}$$

$$\log_2(x+2)+\log_2(8x^3)^{1/3}=4$$
$$\log_2(x+2)+\log_2(2x)=4$$
$$\log_2(2x^2+4x)=4$$

$$2^4=2x^2+4x$$
$$\frac{2x^2}{2}+\frac{4x}{2}-\frac{16}{2}=\frac{0}{2}$$

$$x^2+2x-8=0$$
$$(x+4)(x-2)=0$$
$$x=\cancel{-4},\ 2$$

extraneous

4. A truck is purchased for $51 000 and depreciates by 23% annually. At the same time a minivan is purchased for $38 000 and depreciates by 17% annually. Write an equation to model this situation and use it to determine when the two vehicles will be of equal value.

Truck

t	0	1
V	51000	

$\times 0.77$

$$V = 51000(0.77)^t$$

Van

t	0	1
V	38000	

0.83

$$V = 38000(0.83)^t$$

$$\log\left(51000(0.77)^t\right) = \log\left(38000(0.83)^t\right)$$

$$\log 51000 + t\log 0.77 = \log 38000 + t\log 0.83$$

$$t(\log 0.77 - \log 0.83) = \log 38000 - \log 51000$$

$$t = 3.92 \text{ years}$$

Warm-up...

Evaluate the ~~following without a calculator~~...

$$\boxed{36^{\left(0.5+\log_6 \sqrt{2}\right)}}$$

Clue#1:

$x^{a+b} = (x^a)(x^b)$

Clue#2:

$b^{\log_b x} = x$

$\log_6 2^{1/2}$

$(36)^{0.5}(36)^{\log_6 \sqrt{2}}$

$(\sqrt{36})\ (36)^{\frac{1}{2}\log_6 2}$

$(6)\ \left(36^{\frac{1}{2}}\right)^{\log_6 2}$

$(6)(6)^{\log_6 2}$

$(6)(2)$

12

$36^{\log_6 \sqrt{2}}$

$(6^2)^{\log_6 \sqrt{2}}$

$6^{2\log_6 \sqrt{2}}$

$6^{\log_6 (\sqrt{2})^2}$

$6^{\log_6 2}$

$=2 \times 6 = 12$

Formula:

Change of Base:

$$\log_b M = \frac{\log_x M}{\log_x b}$$

$$\log_3 147 = \frac{\log 147}{\log 3}$$

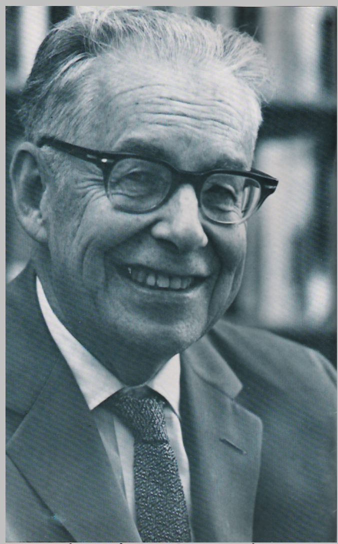
$$= 4.54$$

Logarithmic Scales

I. Richter Scale: Severity of Earthquakes

$$R = \log_{10}\left(\frac{I}{I_o}\right)$$

R - magnitude of an earthquake
I - Intensity of the earthquake
(amplitude of the wave on a seismograph)
I_o - Intensity of the reference earthquake
(1 micron) (1 micron = 10^{-4} cm)



Charles F. Richter
1900-1985

The intensity of the earthquake is measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake.

This implies that an earthquake that reads a 5 on the Richter scale would be 10 times more intense than an earthquake that reads a 4 on the Richter Scale.(The scale jumps by powers of 10)

How many times more intense would an earthquake that reads an 8 on the Richter Scale be than a 5 on the Richter Scale?

If the intensity of earthquake A is 5 and the intensity of earthquake B is 650, what is the difference in their magnitudes as measured by the Richter Scale?

A/ $R = \log\left(\frac{I}{10^{-4}}\right)$

$R = \log\left(\frac{5}{10^{-4}}\right)$

$R = 4.699$

B/ $R = \log\left(\frac{650}{10^{-4}}\right)$

$R = 6.813$

$\underline{\text{Diff.} = 2.114}$

The 1985 Mexico City earthquake had a magnitude of 8.1 on the Richter scale and the 1976 Tangshan earthquake was 1.26 as intense. What was the magnitude of the Tangshan earthquake on the Richter Scale?

8.2

$8.1 = \log\left(\frac{I}{10^{-4}}\right)$

$10^{-4} \cdot 10^{8.1} = \frac{I}{10^{-4}}(10^{-4})$

$I = 10^{4.1} \Longrightarrow$ Tangshan: $I = 1.26(10^{4.1})$

$R = \log\left(\frac{1.26(10^{4.1})}{10^{-4}}\right)$

$\underline{R = 8.2}$