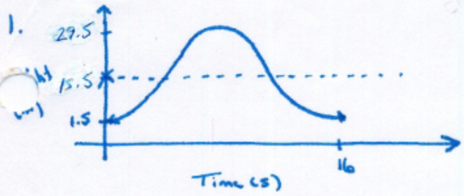
nusoidal Relations : Applications $\boxed{\text{SOLUTIONS}}$

1.



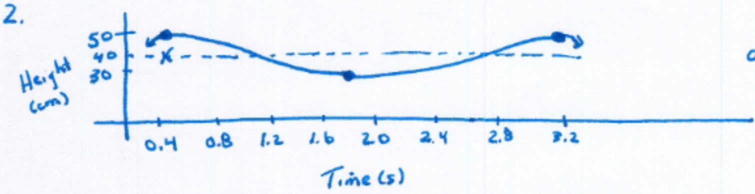

$h = -14\cos 22.5t + 15.5$

At $t = 67s$

$h = -14\cos[22.5 \times 67] + 15.5$

$= \boxed{10.14\ m}$

2.




$h = 10\cos\left(\frac{360}{2.8}\right)(t-0.4) + 40$

a) At $t = 17.2$

$h = 10\cos\left[\left(\frac{360}{2.8}\right)(17.2-0.4)\right] + 40$

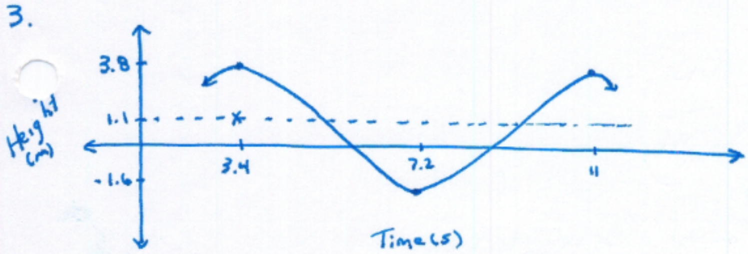
$\boxed{h = 50cm}$

b) At $t = 0s$

$h = 10\cos\left[\left(\frac{360}{2.8}\right)(-0.4)\right] + 40$

$\boxed{h = 46.23cm}$

3.




$h = 2.7\cos\left(\frac{360}{7.6}\right)(t-3.4) + 1.1$

At $t = 40s$

$h = 2.7\cos\left[\left(\frac{360}{7.6}\right)(40-3.4)\right] + 1.1$

$\boxed{h = 2.18\ m}$

4. a) $T = 2.5\sin\left(\frac{360}{365}\right)(n-80) + 12.7$

b) $T = 2.5\sin\left[\left(\frac{360}{365}\right)(163-80)\right] + 12.7$

$\boxed{\begin{array}{l} T = 20.17 \\ \text{So } 8:10 \text{ pm} \end{array}}$

c) 6:30 pm → 18.5 h

$18.5 = 2.5\sin\left[\left(\frac{360}{365}\right)(n-80)\right] + 17.7$

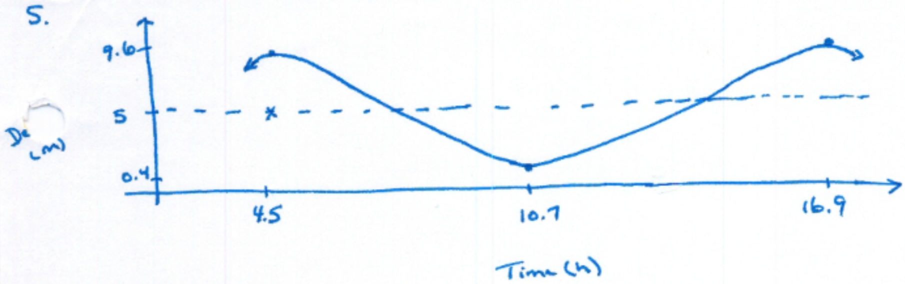
$0.8 = 2.5\sin\left[\left(\frac{360}{365}\right)(n-80)\right]$

$0.32 = \sin\left[\left(\frac{360}{365}\right)(n-80)\right]$

$\left(\frac{365}{360}\right)\sin^{-1}(0.32)_{+80} = n$

$\boxed{\begin{array}{l} n = 99 \\ \text{so April 9th} \end{array}}$

5.



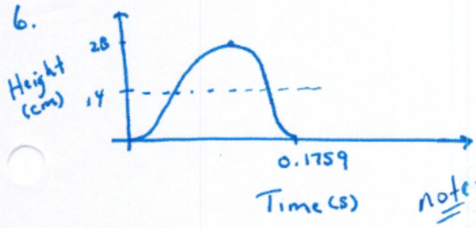

$d = 4.6\cos\left(\frac{360}{12.4}\right)(t-4.5)+5$

At $t = 9.5$

$d = 4.6\cos\left[\left(\frac{360}{12.4}\right)(9.5-4.5)\right]+5$

$\boxed{d = 1.22\ m}$

6.




note: $S = 500\,cm/s$

$C = 2\pi(14)$
$= 28\pi\ cm$
$= 28\pi \times 10^{-5}\ km$

$t = \dfrac{28\pi \times 10^{-5}\ km \cdot h}{18\ km}$
$= 4.8869 \times 10^{-5} h$

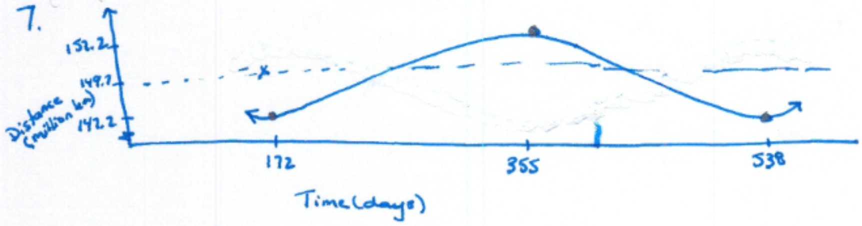
$t = 0.1759\,s$ ← leave in calc! (keep accurate)

$h = -14\cos\left(\frac{360}{0.1759}\right)t + 14$

At $t = 300$, $h = -14\cos(360/0.1759)(300)+14$

$\boxed{h = 12.38\ cm}$

7.




$d = -2.5\cos\left(\frac{360}{365}\right)(t-172)+149.7$

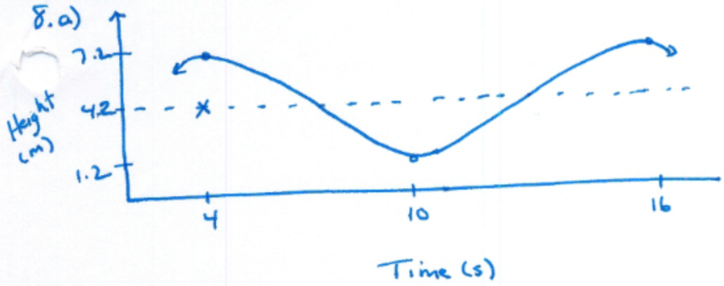
At $t = 260$, $d = -2.5\cos\left[\left(\frac{360}{365}\right)(260-172)\right]+149.7$

$\boxed{d = 149.56\ \text{million km}}$

5.

3/3

8. a)




b) $h = 3\cos 30(t-4) + 4.2$

At $t = 7s$

$h = 3\cos[30(7-4)] + 4.2$

$\boxed{h = 4.2m}$

Solve the following trigonometric equations:

$-12.566$

$$-2.8\sin\left(\frac{\pi}{6}(x-12)\right)+16=17,$$

$$-4\pi \le x \le 4\pi$$

$12.566$

$K=\frac{\pi}{6}$

$Per=\frac{2\pi}{\left(\frac{\pi}{6}\right)}=2\pi\left(\frac{6}{\pi}\right)=12$

$\sin\left(\frac{\pi}{6}(x-12)\right)=\frac{17-16}{-2.8}$

$\sin^{-1}\left(\sin\left(\frac{\pi}{6}(x-12)\right)\right)=\sin^{-1}(\cancel{-}0.357)$

$\theta$

$\pi+\theta$ | $2\pi-\theta$

x | x

Ref∢ $=0.365$

Q3

$\frac{\pi}{6}(x-12)=3.507$

$x=\frac{3.507(6)}{\pi}+12$

$x=18.698$

$-12$

$x=6.698$

$x=-5.302$

$Q3=\pi+0.365$

$Q3=3.507$

$Q4=2\pi-0.365$

$=5.918$

Q4

$\frac{\pi}{6}(x-12)=5.918$

$x=\frac{6(5.918)}{\pi}+12$

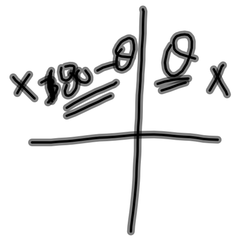
$x=23.303$ $-12K$

$x=11.303$

$x=-0.697$

$6.2 \sin(4(x + 8°)) - 1 = 4,$

$0° \le x \le 360°$




$\sin^{-1}\sin(4(x+8)) = \sin^{-1}\left(\frac{5}{6.2}\right)$

$4(x+8) = \sin^{-1}\left(\frac{5}{6.2}\right)$

Q1 → $\theta = 53.75$

Q2 → $\theta = 180 - 53.75$

$= 126.25$

$4(x+8) = 53.75...$

$x = \frac{53.75}{4} - 8$

$x = 5.44$

$x = 95.44$

$x = 185.44$

$x = 275.44$

$4(x+8) = 126.25$

$x = 23.56 \; + \underline{90°}$

$x = 113.56$

$x = 203.56$

$x = 293.56$

Period $= \frac{360°}{k} = \frac{360}{4} = \underline{90°}$

What about graphs of other trigonometric functions ???

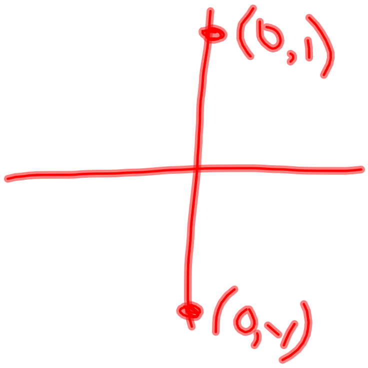
## Graph the Tangent Function

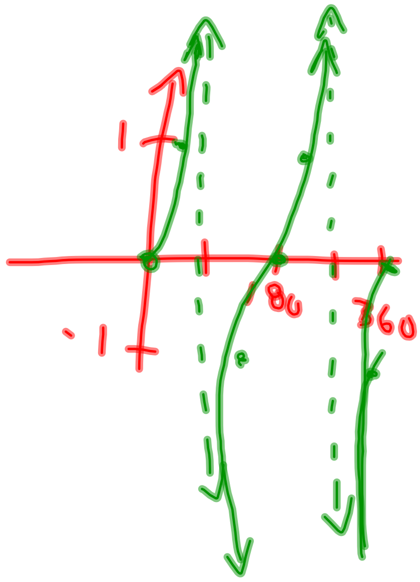
Graph the function $y = \tan\theta$ for $-2\pi \le \theta \le 2\pi$. Describe its characteristics.

| Angle Measure | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| $y$-coordinate on Tangent Line | 0 | 1 | E | -1 | 0 | 1 | E | -1 | 0 |

Asymptotes

$\tan\theta = \frac{y}{x}$



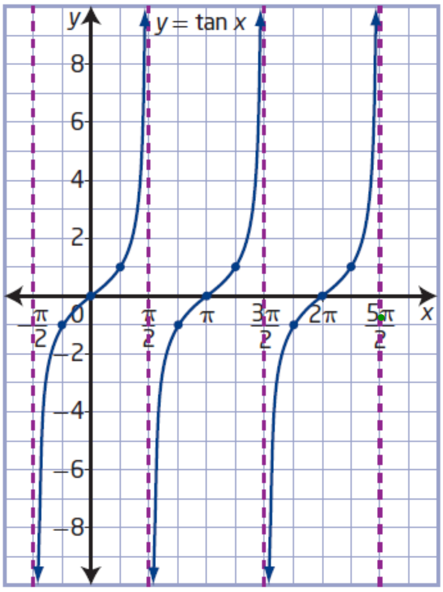



## Key Ideas

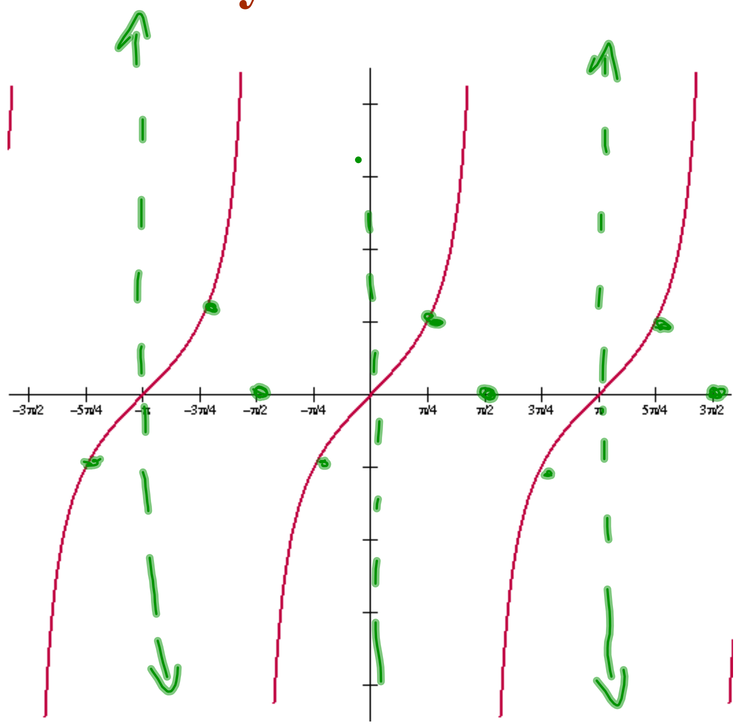
- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph $y = \tan x$, draw one asymptote; draw the points where $y = -1$, $y = 0$, and $y = 1$; and then draw another asymptote.

How can you determine the location of the asymptotes for the function $y = \tan x$?

- The tangent function $y = \tan x$ has the following characteristics:
  - The period is $\pi$.
  - The graph has no maximum or minimum values.
  - The range is $\{y \mid y \in R\}$.
  - Vertical asymptotes occur at $x = \frac{\pi}{2} + n\pi$, $n \in I$.
  - The domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in R, n \in I\right\}$.
  - The $x$-intercepts occur at $x = n\pi$, $n \in I$.
  - The $y$-intercept is 0.



## y = tan θ



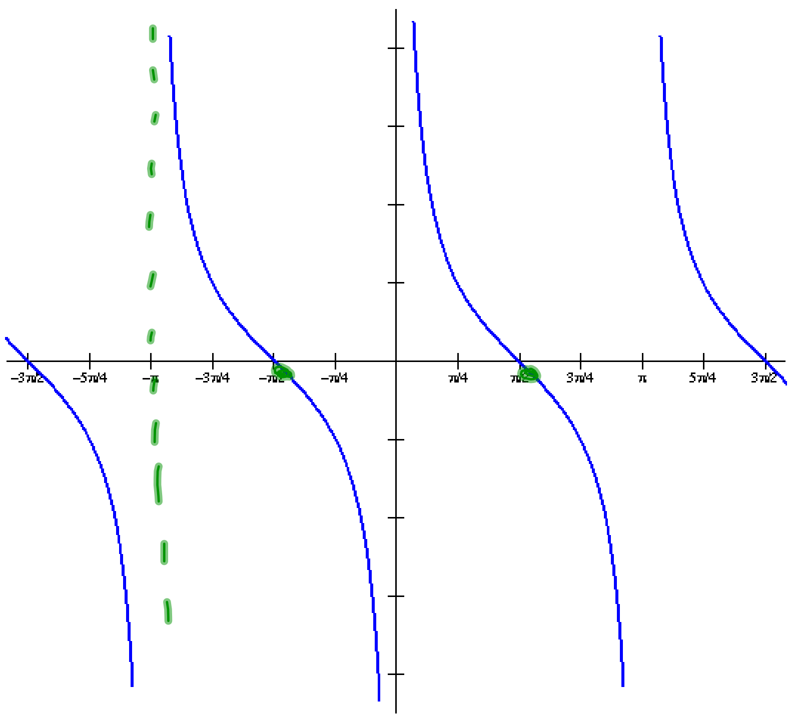
What would the graph of **cot θ** look like?

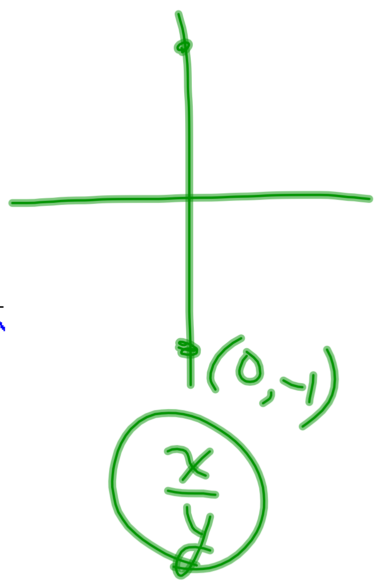
**REMEMBER:**

$$\tan x = \frac{1}{\cot x}$$

where tan x = 0,
cot x is undefined

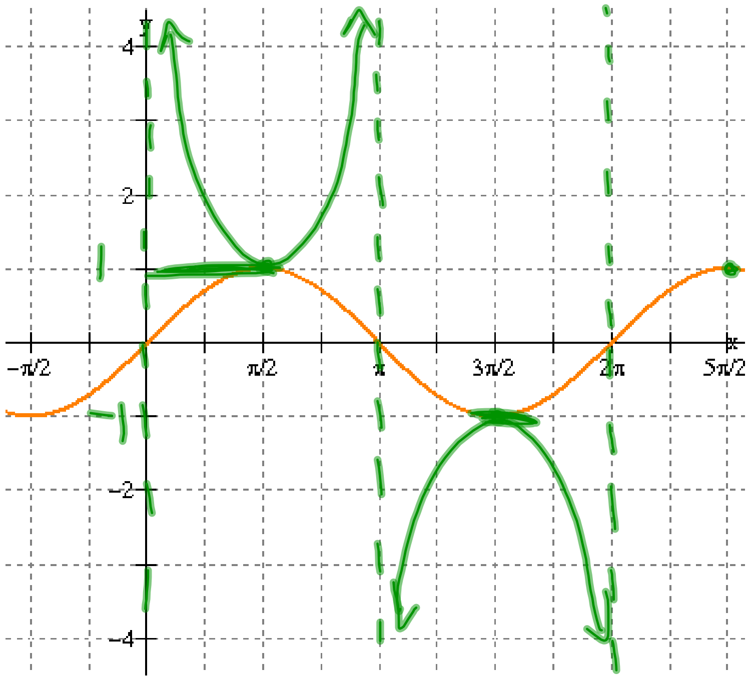
## y = cot θ





# Graphs of Other Trigonometric Functions

y = sin θ




What would the graph of **csc θ** look like?

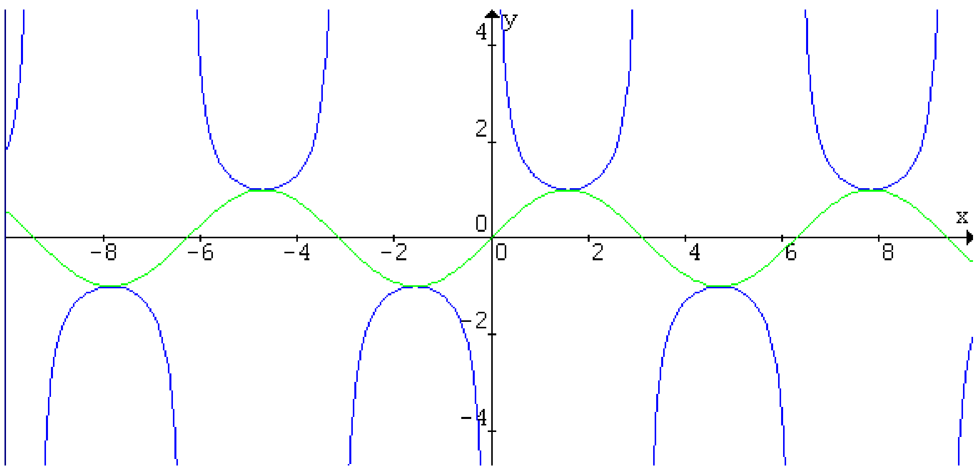
**REMEMBER:**

$$\csc\theta = \frac{1}{\sin\theta}$$

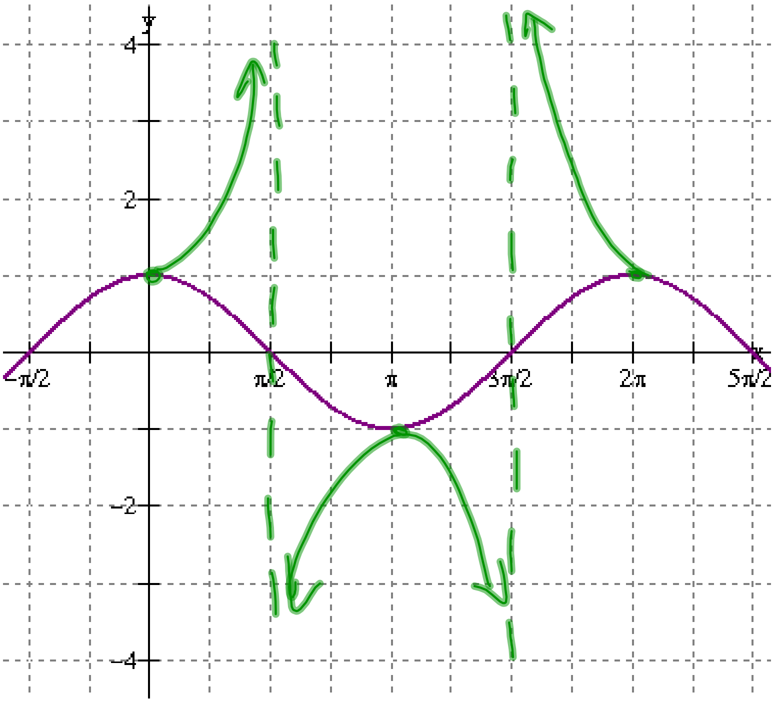
where sin x = 0,
csc x is undefined

y = sin x

y = csc x



## $y = \cos\theta$




What would the graph of **sec θ** look like?

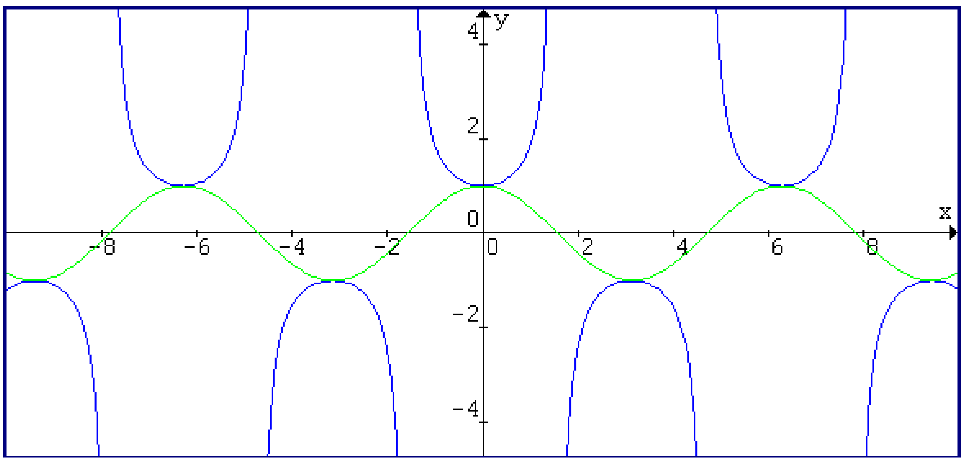
**REMEMBER:**

$$\sec\theta = \frac{1}{\cos\theta}$$

where cos x = 0,
sec x is undefined

y = cos x

y = sec x



## REVIEW - Sketching Trigonometric Functions

- sinusoidal functions
  - properties: domain/range, amplitude, period, phase shift, vertical translation, eq'n of sinusoidal axis, mapping notation.
  - sketching equation in standard form.
- finding the function (both a sine/cosine) given a graph
- solving trigonometric equations where period is not 360
- applications of sinusoidal functions.
  - sketch
  - develop a function
  - use function to answer question
- sketches of all SIX trigonometric ratios