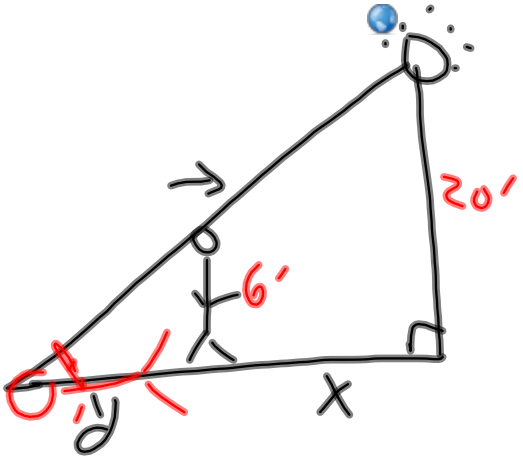
Example 5:

A man 6 feet tall is walking toward a lamppost 20 feet high at a rate of 5 feet per second. The light at the top of the lamppost (20 feet above the ground) is casting a shadow of the man. At what rate is the tip of his shadow moving and at what rate is the length of his shadow changing when he is 10 feet from the base of the lamppost?

animation



$\frac{dx}{dt} = -5 \text{ ft./sec}$

$\frac{6}{20} = \frac{y}{x+y}$

$6x + 6y = 20y$

$6x = 14y$

$6\frac{dx}{dt} = 14\frac{dy}{dt}$

$6(-5) = 14\frac{dy}{dt}$

$\frac{-30}{14} = \frac{dy}{dt}$

$-2.14 \text{ ft./sec} = \frac{dy}{dt}$

Tip of shadow moving ??

$= 7.14 \text{ ft./sec}$ ($5 \text{ ft./s} + 2.14 \text{ ft./sec}$)

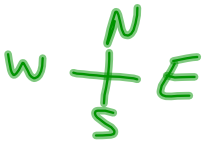
Warm Up

At 9 A.M. ship A is situated 80 *km* due east of ship B. Ship A is traveling north at 40 *km/h* and ship B is sailing south at 60 *km/h*. How fast is the distance between the ships changing at noon ?

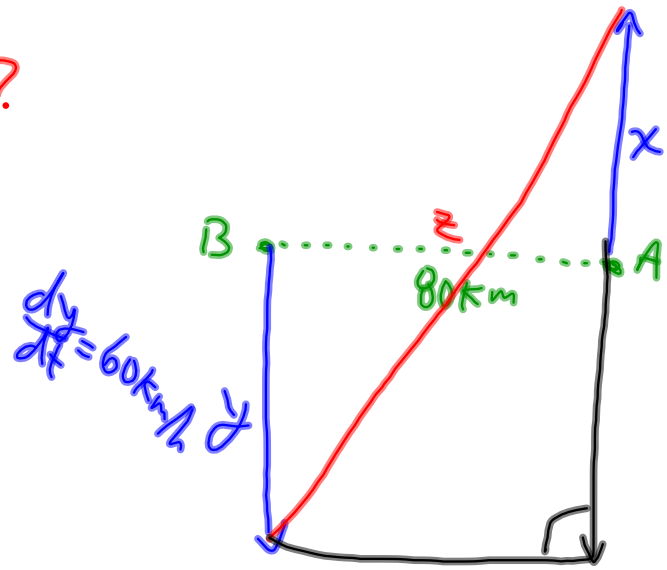
A building is illuminated by a floodlight that is 15 m away and at ground level a man 2 m tall walks away from the light directly towards the building at 2 m/s. Determine the rate of change of the length of his shadow when he is 4 m from the light ? [5]

The trough down the centre of a cattle barn is 40 cm wide at the top and 20 cm at the bottom. It is 30 cm deep and 8 m long. The trough is being filled at the rate of 0.25 m^3/min. How fast is the water level in the trough rising when the water is 20 cm deep in the trough?

At 9 A.M. ship A is situated 80 *km* due east of ship B. Ship A is traveling north at 40 *km/h* and ship B is sailing south at 60 *km/h*. How fast is the distance between the ships changing at noon ?

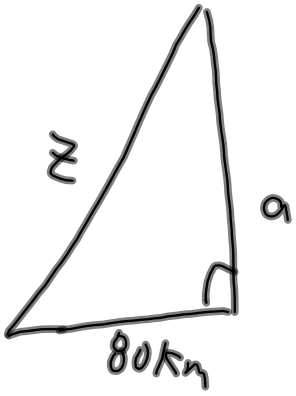


$\frac{dz}{dt} = ?$



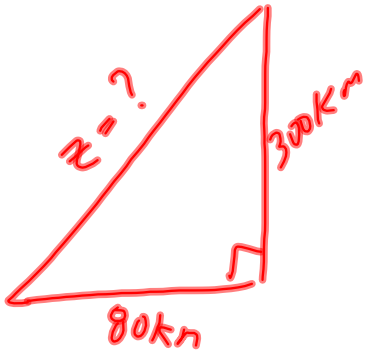
$\frac{dx}{dt} = 40 km/h$

$\frac{dy}{dt} = 60 km/h$



$\frac{da}{dt} = 100 km/h$

At Noon . . .



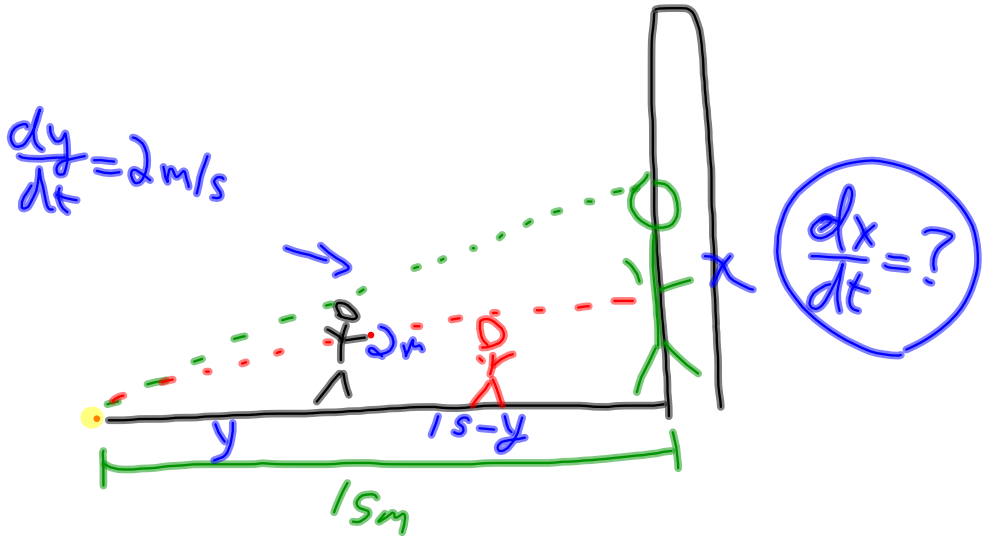
$a^2 + 80^2 = z^2$

$2a\frac{da}{dt} = 2z\frac{dz}{dt}$

$2(300)(100) = 2\sqrt{300^2 + 80^2}\,\frac{dz}{dt}$

$\frac{dz}{dt} = 96.6 km/h$

A building is illuminated by a floodlight that is 15 m away and at ground level a man 2 m tall walks away from the light directly towards the building at 2 m/s. Determine the rate of change of the length of his shadow when he is 4 m from the light ? [5]



Similar Triangles

$$\frac{x}{2}=\frac{15}{y}$$

$$xy=30$$

$$\frac{dx}{dt}y+x\frac{dy}{dt}=0$$

$$\frac{dx}{dt}(4)+7.5(2)=0$$

$$\frac{dx}{dt}=\frac{-15}{4}=-3.75\,m/s$$

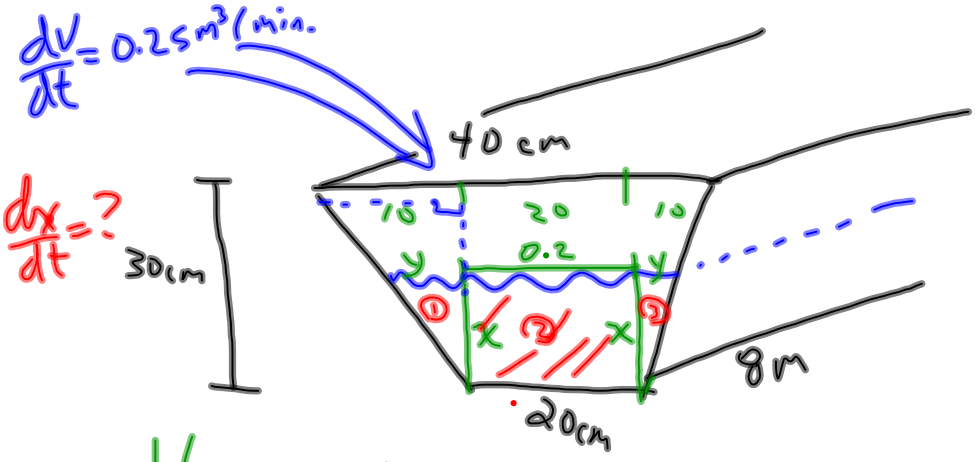
$y=4,\ \frac{dy}{dt}=2$

$xy=30$

$x(4)=30$

$x=\frac{30}{4}=7.5$

The trough down the centre of a cattle barn is 40 cm wide at the top and 20 cm at the bottom. It is 30 cm deep and 8 m long. The trough is being filled at the rate of 0.25 m^3/min. How fast is the water level in the trough rising when the water is 20 cm deep in the trough?

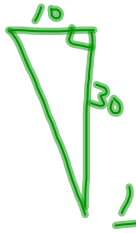


$$V_{PRISM} = (\text{Area of Face}) \times \text{length}$$

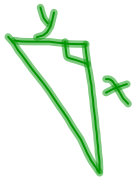
$$V = \left[\left(\frac{xy}{2}\right)2 + 0.2x\right](8)$$

$$V = 8x\left(\frac{1}{3}x\right) + 1.6x \quad \text{eliminate variable "y"}$$

Trough



Water



$$\frac{10}{y} = \frac{30}{x}$$

$$10x = 30y$$

$$\frac{10}{30}x = y$$

$$\frac{1}{3}x = y$$

$$V = \frac{8}{3}x^2 + 1.6x$$

$$\frac{dV}{dt} = \frac{16}{3}x\frac{dx}{dt} + 1.6\frac{dx}{dt}$$

$$0.25 = \frac{16}{3}(0.2)\frac{dx}{dt} + 1.6\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{0.25}{\left(\frac{3.2}{3} + 1.6\right)} = 0.094 \text{ m/minutes}$$

$$= 9.4 \text{ cm/minute}$$

① Finish Pg. 145/146 #3 – 17 (omit #14)

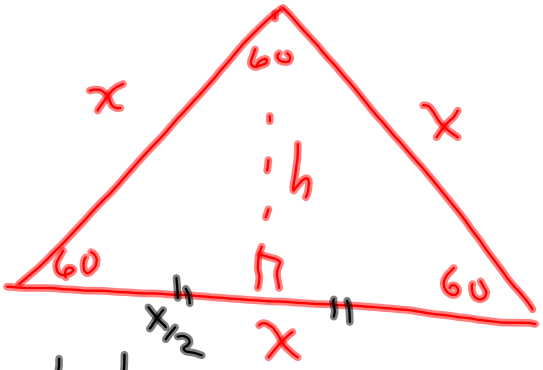
② Worksheet

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

8/



$A = \frac{1}{2}xh$

$\sin 60° = \frac{h}{x}$

$h = x \sin 60°$

$h = \frac{\sqrt{3}}{2}x$

$A = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right)$

$A = \frac{\sqrt{3}}{4}x^2$

$A = \frac{1}{2}bh$

OR

$A = \frac{1}{2}x^2 \sin 60°$

$A = \frac{1}{2}x^2\left(\frac{\sqrt{3}}{2}\right)$

$A = \frac{\sqrt{3}}{4}x^2$