Warm Up

A rock is thrown into a pool of water. A circular wave leaves the point of impact and travels so that its radius increases at a rate of 25 cm/s. How fast is the circumference of the wave increasing when the radius of the wave is 1m ?

$\frac{dr}{dt} = 25 cm/s$ $\frac{dC}{dt} = ?$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dC}{dt} = 2\pi(25)$

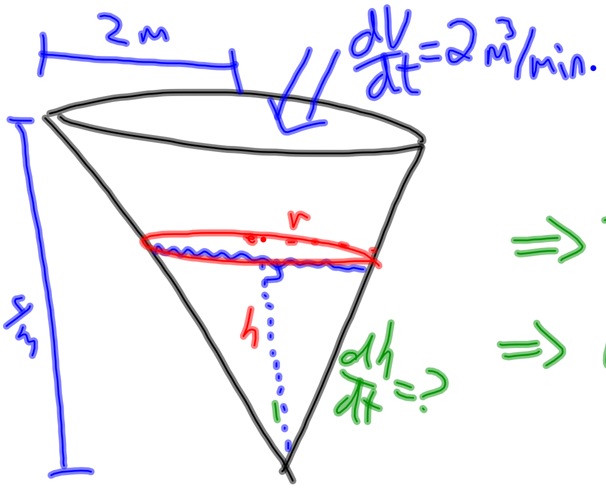
$= 50\pi\, cm/sec$

$y = 6x$

$\frac{dy}{dx} = 6(1)$

Example:

A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at 2 m³/min. , find the rate at which the water level is rising when the water is 3 m deep.

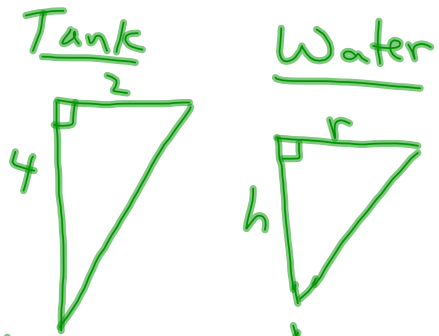


$$V = \frac{1}{3}\pi r^2 h$$

⇒ Too Many unknowns!!

⇒ Would like to get "r" in terms of "h".

Compare the tank to the water . . .



Use similar △s . . .

$$\frac{4}{h} = \frac{2}{r}$$

$$4r = 2h$$

$$r = \frac{2}{4}h$$

$$r = \frac{1}{2}h$$

⇒ Re-construct Formula . . .

$$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$2 = \frac{1}{4}\pi(3)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8}{9\pi}\ m/min.$$

$$= 0.28\ m/min$$

Example 4:

Water is being poured into a conical reservoir at the rate of pi cubic feet per second. The reservoir has a radius of 6 feet across the top and a height of 12 feet. At what rate is the depth of the water increasing when the depth is 6 feet?

animation

The volume of the water in the reservoir (V) is given by

$V = \frac{1}{3}\pi r^2 h.$

$$\frac{6}{r} = \frac{12}{h}$$

$$6h = 12r$$

$$r = \frac{1}{2}h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$\pi = \frac{\pi}{4}(6)^2 \frac{dh}{dt}$$

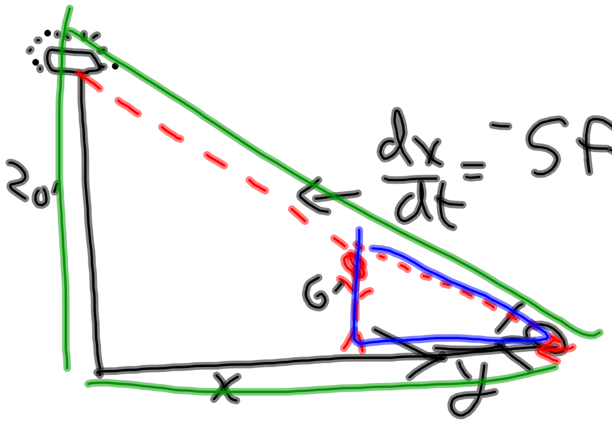
$$\frac{4\cancel{\pi}}{36\cancel{\pi}} = \frac{dh}{dt}$$

$$\frac{1}{9} \text{ ft./sec} = \frac{dh}{dt}$$

Example 5:

A man 6 feet tall is walking toward a lamppost 20 feet high at a rate of 5 feet per second. The light at the top of the lamppost (20 feet above the ground) is casting a shadow of the man. At what rate is the tip of his shadow moving and at what rate is the length of his shadow changing when he is 10 feet from the base of the lamppost?

animation



$\frac{dx}{dt} = {}^{-}5 \text{ ft./s}$

$\frac{6}{20} = \frac{y}{x+y}$

$6x + 6y = 20y$

$6x = 14y$

$6\frac{dx}{dt} = 14\frac{dy}{dt}$

$6(-5) = 14\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{-30}{14} \text{ ft./sec}$

(i) Shadow is moving at

$5 \text{ ft./s} + 2.14 \text{ ft./s}$

(ii) $\frac{dy}{dt} = {}^{-}2.14 \text{ ft./sec}$

Warm Up

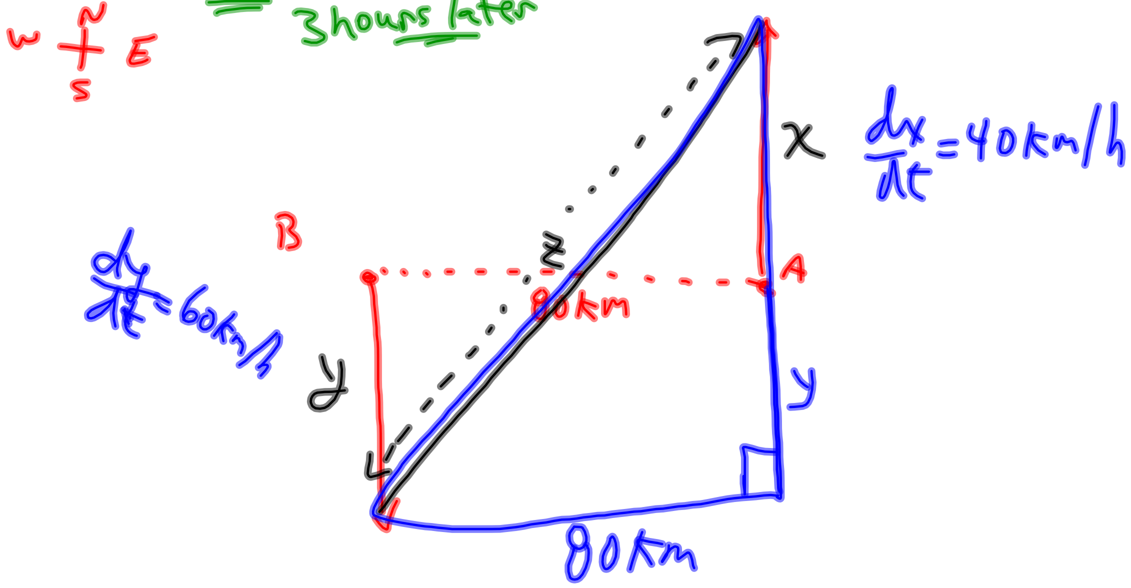
At 9 A.M. ship A is situated 80 *km* due east of ship B. Ship A is traveling north at 40 *km/h* and ship B is sailing south at 60 *km/h*. How fast is the distance between the ships changing at noon ?

A building is illuminated by a floodlight that is 15 m away and at ground level a man 2 m tall walks away from the light directly towards the building at 2 m/s. Determine the rate of change of the length of his shadow when he is 4 m from the light ? [5]

The trough down the centre of a cattle barn is 40 cm wide at the top and 20 cm at the bottom. It is 30 cm deep and 8 m long. The trough is being filled at the rate of 0.25 m^3/min. How fast is the water level in the trough rising when the water is 20 cm deep in the trough?

At 9 A.M. ship A is situated 80 *km* due east of ship B. Ship A is traveling north at 40 *km/h* and ship B is sailing south at 60 *km/h*. How fast is the distance between the ships changing at noon ?

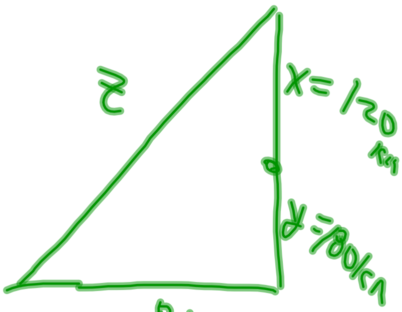
= 3 hours later



$$(x+y)^2 + 80^2 = z^2$$

At Noon ...

$$2(x+y)^1\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2z\frac{dz}{dt}$$



$$2(300)(100) = 2\sqrt{80^2+300^2}\,\frac{dz}{dt}$$

$$\sqrt{80^2+300^2} = z$$

$$\frac{dz}{dt} = \frac{2(300)(100)}{2\sqrt{80^2+300^2}}$$

$$= 96.6 \text{ km/h}$$