Warm Up

Differentiate the following...

$$\left(x^3 + y^5\right)^6 + 3xy = 2x^4y^5$$

$$6(x^3+y^5)^5\left(3x^2+5y^4\frac{dy}{dx}\right)+3y+3x\frac{dy}{dx}=8x^3y^5+2x^4\left(5y^4\frac{dy}{dx}\right)$$

$$18x^2(x^3+y^5)^5+30y^4(x^3+y^5)^5\frac{dy}{dx}+3y+3x\frac{dy}{dx}=8x^3y^5+10x^4y^4\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx}=\frac{8x^3y^5-18x^2(x^3+y^5)^5-3y}{30y^4(x^3+y^5)^5+3x-10x^4y^4}}$$

Higher Order Derivatives

We can continue to find the derivatives of a derivative. We find the

- second derivative by taking the derivative of the first,
- third derivative by taking the derivative of the second ... etc

Examples:

1. Determine the higher order derivatives for *f(x)*...

$$f(x) = x^4 - 2x^3 + 3x - 5$$

$f'(x) = 4x^3 - 6x^2 + 3$

$f''(x) = 12x^2 - 12x$

$f'''(x) = 24x - 12$

$f''''(x) = 24$

$f'''''(x) = \underline{\underline{0}}$

$f^{(5)}$

2. Determine $f'''(x)$ given that $f(x) = \dfrac{5}{\sqrt{2-3x}}$

$f(x) = 5(2-3x)^{-\frac{1}{2}}$

$f'(x) = -\frac{5}{2}(2-3x)^{-3/2}(-3)$

$f'(x) = \frac{15}{2}(2-3x)^{-3/2}$

$f''(x) = -\frac{45}{4}(2-3x)^{-5/2}(-3)$

$f''(x) = \frac{135}{4}(2-3x)^{-\frac{5}{2}}$

$f'''(x) = -\frac{675}{8}(2-3x)^{-7/2}(-3)$

$f'''(x) = \frac{2025}{8}(2-3x)^{-7/2}$

3. Find the second derivative of the implicit function $xy + y^2 = 4$.

$$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

y' $\quad$ $\frac{dy}{dx}$

y'' $\quad$ $\frac{d^2y}{dx^2}$

y''' $\quad$ $\frac{d^3y}{dx^3}$

$$\frac{dy}{dx} = \frac{-y}{x+2y}$$

$$\frac{d^2y}{dx^2} = \frac{\left(-\frac{dy}{dx}\right)(x+2y) + y\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(-\frac{-y}{\cancel{x+2y}}\right)\cancel{(x+2y)} + y\left(1+2\left(\frac{-y}{x+2y}\right)\right)}{(x+2y)^2}$$

$$= \left(\frac{y+y}{1} - \frac{2y^2}{x+2y}\right)\cdot\frac{1}{(x+2y)^2}$$

$$= \left[\frac{2y(x+2y) - 2y^2}{x+2y}\right]\cdot\frac{1}{(x+2y)^2}$$

$$= \frac{2y^2+2xy}{(x+2y)^3}$$

4. *Determine the fourth derivative of* $y = \cos(5x)$

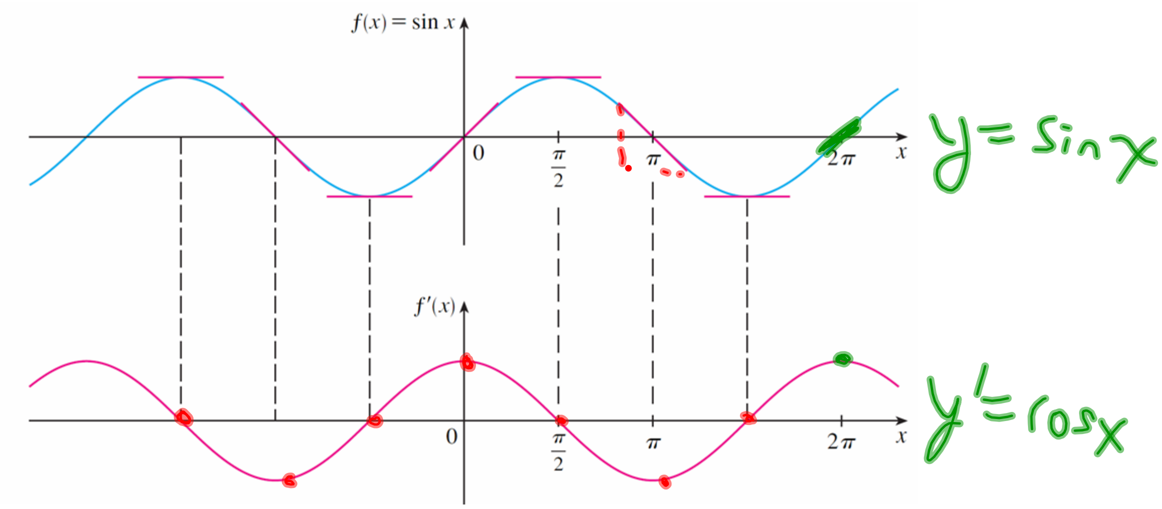
HOMEWORK

Page 111
(Higher Order Derivatives)

#2, 3, 4, 5, 7 (a)

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{(\sin x \cos h + \cos x \sin h) - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\underbrace{\frac{\cos h - 1}{h}}_{=0} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

$\lim_{x\to 3} 17$

$= 17$

$f(x) = \sin x$

$f'(x) = \cos x$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = [\sec^2 u] \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

ex.

$$y = \tan\overbrace{(7x^4)}^{u}$$

$$y' = [\sec^2(7x^4)](28x^3)$$

$$y = \csc\sqrt{3x-5}$$

$$y' = -\csc\sqrt{3x-5}\cot\sqrt{3x-5}\left[\frac{1}{2}(3x-5)^{-1/2}(3)\right]$$