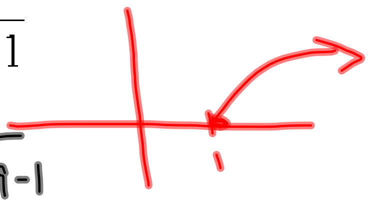
Recall from our prior discussions that ...

1 Theorem $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$

Let's look at a couple of unique functions:

1) $\lim_{x\to 1^+} \sqrt{x-1}$



$\lim_{x\to 1^-} \sqrt{0.999-1}$

$\sqrt{\text{small}(-)}$

DNE

2) $\lim_{x\to -2} \dfrac{|x+2|}{x+2}$

$\lim_{x\to -2^-} \dfrac{|-2.000..1+2|}{-2.00...1+2}$

$= -1$

$\lim_{x\to -2^+} \dfrac{|-1.999..+2|}{-1.999..+2}$

$\dfrac{+}{+}$

$= 1$

Warm Up

Evaluate the following limits...

$$\lim_{x\to -1^-} \frac{|x+1|}{x^2-1}$$

$$\lim_{x\to -1^-} \frac{|x+1|}{(x-1)(x+1)}$$

$$\lim_{x\to -1^-} \frac{\cancel{|-1.00..1+1|}^{-1}}{(-1.00..1-1)(\cancel{-1.00..1+1})}$$

$$= +\frac{1}{2}$$

$$\lim_{x\to 7} \sqrt[4]{x-7} \quad \therefore \text{DNE}$$

$$\lim_{x\to 7^-} \sqrt[4]{6.99.-7}$$

$$\sqrt[4]{-}$$

DNE

Check for any point(s) of discontinuity...

$$f(x)=\begin{cases} -x^2+2, & \text{if } x<1 \\ 1, & \text{if } x=1 \\ -2x+3, & \text{if } 1<x\le 3 \\ -3, & \text{if } x>3 \end{cases}$$

Sketch $f(x)$...

Check for any point(s) of discontinuity...

$$f(x)=\begin{cases} -x^2+2 \quad , & if \;\; x<1 \\ 1 \quad , & if \;\; x=1 \\ -2x+3 \quad , & if \;\; 1<x\leq 3 \\ -3 \quad , & if \;\; x>3 \end{cases}$$

Sketch $f(x)$...

$\underline{x=1}$

$f(1)=1$

$\lim\limits_{x\to 1^-} f(x) = -(1)^2+2 = 1$

$\lim\limits_{x\to 1^+} f(x) = -2(1)+3 = 1$

$\therefore$ continuous at $x=1$

$f(1)=\lim\limits_{x\to 1} f(x)$

$\underline{x=3}$

$f(3) = -2(3)+3 = -3$

$\lim\limits_{x\to 3^-} f(x) = -3$

$\lim\limits_{x\to 3^+} f(x) = -3$

$\therefore$ continuous at $x=3$

$f(3)=\lim\limits_{x\to 3} f(x)$

1/ $y=-x^2+2$

$V(0,2)$

opens down

x	y
1	1

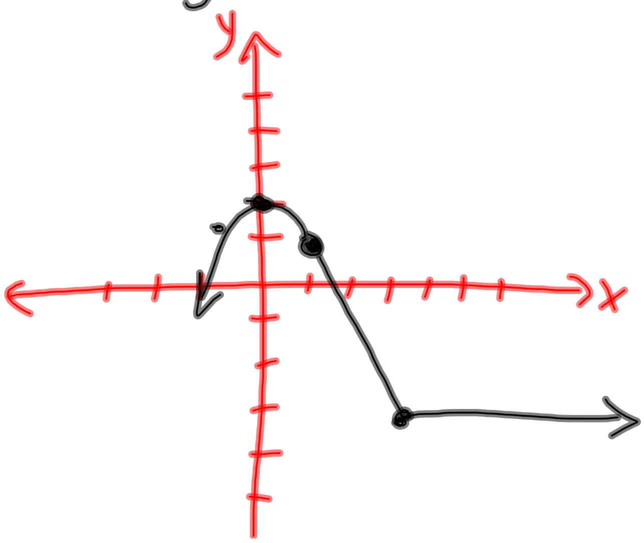
2/ $(1,1)$

3/ $y=-2x+3$

x	y
1	1
3	-3

4/ $y=-3$

$$f(x)=\begin{cases} -x^2+2 \quad , & if \;\; x<1 \\ 1 \quad , & if \;\; x=1 \\ -2x+3 \quad , & if \;\; 1<x\leq 3 \\ -3 \quad , & if \;\; x>3 \end{cases}$$



Warm Up

Given the function...

$$h(x)=\begin{cases}-(x+2)^2+1 & \text{if } x<-1\\ 2 & \text{if } x=-1\\ \sqrt{x+1} & \text{if } -1<x\le 3\\ |x-4|+1 & \text{if } 3<x<5\\ 7-x & \text{if } x>5\end{cases}$$

(1) Examine $h(x)$ for any points of discontinuity. Provide a mathematical reason for any point of discontinuity.

(2) Sketch $h(x)$

Solution:

$x=-1$

$h(-1)=2$

$\lim_{x\to -1^-} h(x) = -(-1+2)^2+1 = 0$

$\lim_{x\to -1^+} h(x) = 0$

$\therefore$ Discontinuous at $x=-1$

$\lim_{x\to -1} h(x) \neq h(-1)$

$x=3$

$h(3)=\sqrt{3+1}=2$

$\lim_{x\to 3^-} h(x) = 2$

$\lim_{x\to 3^+} h(x) = |3-4|+1 = 2$

$\therefore$ continuous at $x=3$

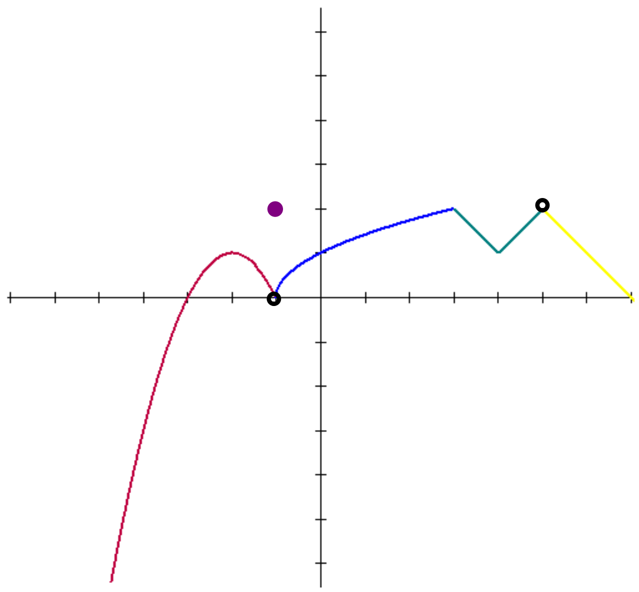
$\lim_{x\to 3} h(x) = h(3)$

$x=5$

$h(5)=$ D.N.E

$\therefore$ discontinuous at $x=5$

$h(5)$ Does Not exist



$y=-(x+2)^2+1$

$V(-2,1)$

opens down

x	y
-1	0

$(-1,2)$

$y=\sqrt{x+1}$

x	y
-1	0
3	2

$y=|x-4|+1$

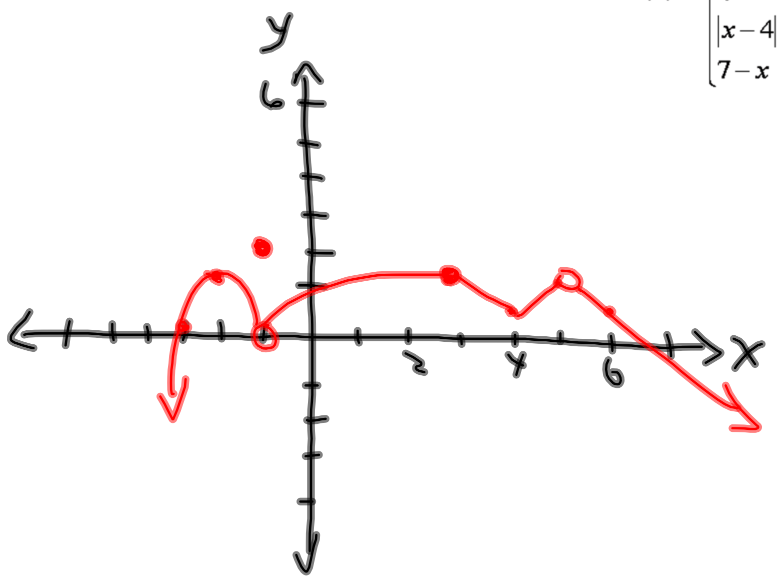
$V(4,1)$

x	y
3	2
5	2

$$h(x)=\begin{cases} -(x+2)^2+1 & \text{if } x<-1 \\ 2 & \text{if } x=-1 \\ \sqrt{x+1} & \text{if } -1<x\le 3 \\ |x-4|+1 & \text{if } 3<x<5 \\ 7-x & \text{if } x>5 \end{cases}$$

$y=7-x$

x	y
5	2
6	1



Homework

Page 27 - 29

1 - 10