Warm-Up...

$$f(x)=\begin{cases}-(x+2)^2+1 & \text{if } x<-1\\ x+1 & \text{if } -1\le x<2\\ 1 & \text{if } x=2\\ 3 & \text{if } x>2\end{cases}$$

(a) Using the three conditions for continuity examine *f(x)* for any points of discontinuity.
At any point(s) of discontinuity clearly indicate a mathematical reason to support why the function is discontinuous at that particular point. **[4]**

(b) Draw a sketch of *f(x)* to support what you have found in part (a). **[4]**

$\underline{x=-1}$

$f(-1)=-1+1$
$=0$

$\lim\limits_{x\to -1^-} f(x)$
$=-(-1+2)^2+1$
$=0$

$\lim\limits_{x\to -1^+} f(x)$
$=0$

$\lim\limits_{x\to -1} f(x)=f(-1)$

$\therefore$ continuous

$\underline{x=2}$

$f(2)=1$

$\lim\limits_{x\to 2^-} f(x)$
$=2+1$
$=3$

$\lim\limits_{x\to 2^+} f(x)$
$=3$

$\lim\limits_{x\to 2} f(x)\neq f(2)$

$\therefore$ discontinuous

$y=-(x+2)^2+1$
$V(-2,1)$
opens down

x	y
-1	0

$y=x+1$

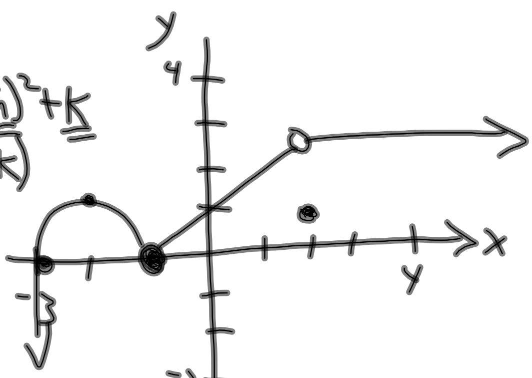
x	y
-1	0
2	3

$(2,1)$

$y=3$

$$f(x)=\begin{cases}-(x+2)^2+1 & \text{if } x<-1\\ x+1 & \text{if } -1\le x<2\\ 1 & \text{if } x=2\\ 3 & \text{if } x>2\end{cases}$$

$y=a(x-h)^2+k$
$V(h,k)$



$y=3x^2-18x+1$
$y=3(x^2-6x+9)+1-27$
$y=3(x-3)^2-26$
$V(3,-26)$

Completing The Square

Limits at Infinity

What exactly is infinity?

- It is the ***process*** of making a value arbitrarily large or small

$+\infty$ ⟶ Positive Infinity...process of becoming arbitrarily large

$-\infty$ ⟶ Negative Infinity...process of becoming arbitrarily small

4 Definition Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

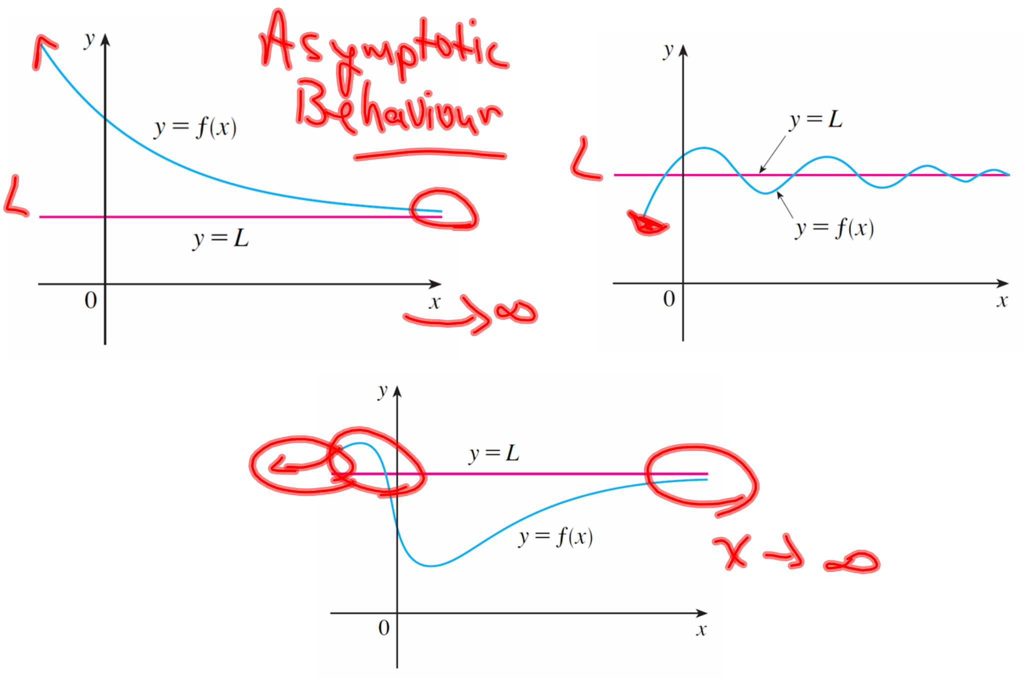


FIGURE 9
Examples illustrating $\lim_{x \to \infty} f(x) = L$

Have a look at these limits...

$$\lim_{x \to \infty} \frac{1}{x}$$

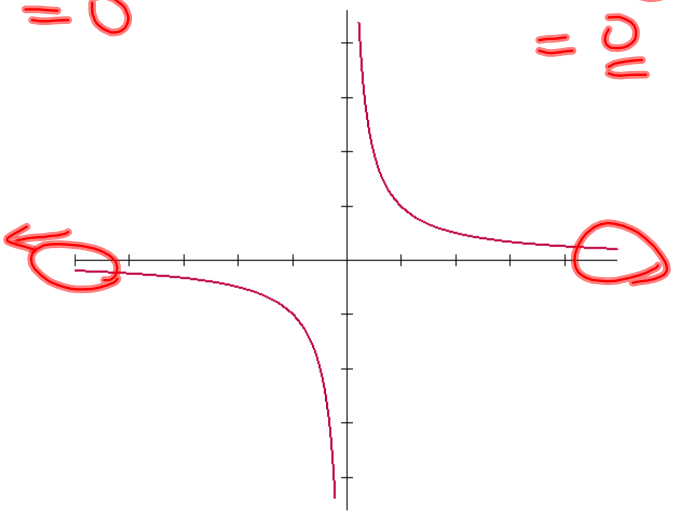
$\to 0^+$

$= 0$

$$\lim_{x \to -\infty} \frac{1}{x}$$

$\to 0^-$

$= 0$



In general...

$\boxed{7}$ If n is a positive integer, then

$$\lim_{x \to \infty} \frac{1}{x^n} = 0 \qquad \lim_{x \to -\infty} \frac{1}{x^n} = 0$$

Calculating limits at infinity without using a graph

• Rational Functions

Note: If every term in a rational expression is divided by the same value, the rational expression will still be equal to it's original value

$\frac{20}{4} = 5$

$$\frac{12+8}{6-2}$$

Divde the numerator and denominator by 2

$$\frac{6+4}{3-1} = \frac{10}{2} = 5$$

This will be important when evaluating limits for rational functions approaching infinity...

Look at the following example:

$$\boxed{\lim_{x\to\infty}\frac{3x^2 - x - 2}{5x^2 + 4x + 1}} = \lim_{x\to\infty}\frac{\dfrac{3x^2 - x - 2}{x^2}}{\dfrac{5x^2 + 4x + 1}{x^2}} = \lim_{x\to\infty}\frac{3 - \dfrac{1}{x} - \dfrac{2}{x^2}}{5 + \dfrac{4}{x} + \dfrac{1}{x^2}}$$

Divide every term by the HIGHEST power that is present in either the numerator or denominator of the rational expression once they are expanded

$$= \frac{\lim_{x\to\infty}\left(3 - \dfrac{1}{x} - \dfrac{2}{x^2}\right)}{\lim_{x\to\infty}\left(5 + \dfrac{4}{x} + \dfrac{1}{x^2}\right)}$$

$$= \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty}\dfrac{1}{x} - 2\lim_{x\to\infty}\dfrac{1}{x^2}}{\lim_{x\to\infty} 5 + 4\lim_{x\to\infty}\dfrac{1}{x} + \lim_{x\to\infty}\dfrac{1}{x^2}}$$

$$= \frac{3 - 0 - 0}{5 + 0 + 0} \qquad \text{[by (7)]}$$

$$= \frac{3}{5}$$

This graph below validates our solution:

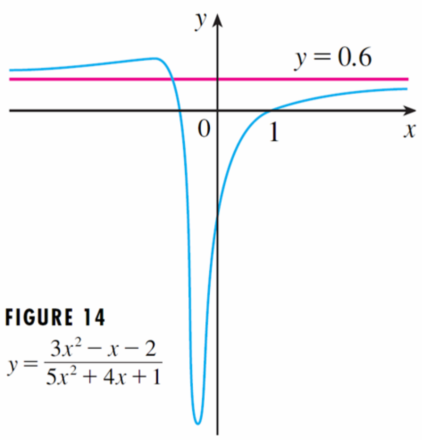


FIGURE 14

$y = \dfrac{3x^2 - x - 2}{5x^2 + 4x + 1}$

$$\lim_{x\to\infty} \frac{(5-x^2)^2}{2x^4+x-5} \leftarrow \text{Not a polynomial!}$$

$$\lim_{x\to\infty} \frac{\frac{25}{x^4}-\frac{10x^2}{x^4}+\frac{x^4}{x^4}}{\frac{2\cancel{x^4}}{\cancel{x^4}}+\frac{x}{x^4}-\frac{5}{x^4}}$$

$$\frac{x^4}{x^4}=1$$

$$=\frac{0-0+1}{2+0-0}$$

$$\lim_{x\to\infty}\frac{1}{x^3}$$

$$=\frac{1}{2}$$

Evaluate the following limit:

$$\lim_{x\to\infty} \frac{-3(x^2-4)^2}{3-5x^2}$$

$$\lim_{x\to\infty} \frac{-3(x^4-8x^2+16)}{3-5x^2}$$

$$\lim_{x\to\infty} \frac{\frac{-3\cancel{x^4}}{\cancel{x^4}} + \underbrace{\frac{24x^2}{x^4}}_{\to 0} - \underbrace{\frac{48}{x^4}}_{\to 0}}{\underbrace{\frac{3}{x^4}}_{\to 0} - \underbrace{\frac{5x^2}{x^4}}_{\to 0}}$$

$$= \frac{-3+0-0}{0-0} = \frac{-3}{0} \quad \underline{\text{undefined}}$$

$\therefore$ Does Not Exist

$$\lim_{x\to\infty} \frac{(3x^3-x^4)(1-x^6)}{(2-3x^5)^2}$$

$$\lim_{x\to\infty} \frac{\frac{3x^3}{x^{10}} - \frac{3x^9}{x^{10}} - \frac{x^4}{x^{10}} + \frac{x^{10}}{x^{10}}}{\frac{4}{x^{10}} - \frac{12x^5}{x^{10}} + \frac{9\cancel{x^{10}}}{\cancel{x^{10}}}}$$

$$= \frac{0-0-0+1}{0-0+9}$$

$$= \frac{1}{9}$$

• Exponential Functions

$e \Rightarrow$ Euler's #

$\lim_{x\to\infty} e^x$

$\to \infty$

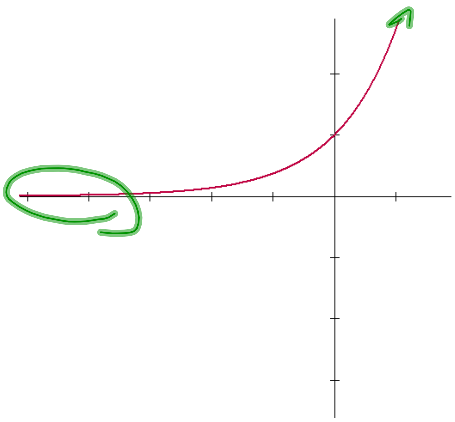
$\therefore$ DNE

$\lim_{x\to-\infty} e^x = 2.718\ldots$

$= 2.718^{-\infty}$

$= \dfrac{1}{2.718\ldots^{\infty}} = \dfrac{1}{\infty}$

$= 0$



Try each of these...

1. $\lim_{x\to-\infty} 2^{3-x}$

$\lim_{x\to-\infty} 2^{3-(-\infty)}$

$= 2^{3+\infty}$

$= 2^{\infty}$

$\to \infty$

$\therefore$ DNE

2. $\lim_{x\to\infty} \dfrac{2^x}{5^x}$

$= \left(\dfrac{2}{5}\right)^x = \dfrac{2^\infty}{5^\infty}$

$(<1)^\infty \to 0$

$= 0$

3. $\lim_{x\to\infty} \dfrac{3^x}{4}$

$= \dfrac{3^\infty}{4} = \dfrac{\infty}{4}$

$\to \infty$

D.N.E

4. $\lim_{x\to-\infty} \dfrac{1}{5^x}$

$= \dfrac{1}{5^{-\infty}}$

$= \dfrac{5^\infty}{1} \to \infty$

D.N.E

Homework:

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