Warm Up

Evaluate each of the following if they exist...

$$\lim_{x \to \infty} \frac{\left(2 - 3x^2\right)^2}{\left(2x^2 + 1\right)\left(3x^2 - 5\right)} \qquad \boxed{\frac{3}{2}}$$

$$\lim_{x \to \infty} \frac{\dfrac{4}{x^4} - \dfrac{12x^4}{x^4} + \dfrac{9x^4}{x^4}}{\dfrac{6x^4}{x^4} - \dfrac{7x^2}{x^4} - \dfrac{5}{x^4}} = \frac{0 - 0 + 9}{6 - 0 - 0} = \frac{9}{6} = \boxed{\frac{3}{2}}$$

$\boxed{\dfrac{\sqrt{2}}{128}}$

$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x^4 - 16}$$

$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{(x^2 - 4)(x^2 + 4)}$$

$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{(x - 2)(x + 2)(x^2 + 4)}$$

$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{(\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})(x + 2)(x^2 + 4)}$$

$$= \frac{1}{(2\sqrt{2})(4)(8)}$$

$$= \frac{1}{64\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \boxed{\frac{\sqrt{2}}{128}}$$

$$\lim_{x \to -5^+} \frac{|x + 5|}{4x + 20} \qquad \boxed{\frac{1}{4}}$$

$$\lim_{x \to -5^+} \frac{|x + 5|}{4(x + 5)}$$

$$\lim_{x \to -5^+} \frac{|-4.99...+5|^{\,1}}{4(-4.99...+5)}$$

$$= \frac{1}{4}$$

1

Pg. 50

#3(i) $\lim\limits_{n\to\infty} \dfrac{(-1)^{n+1}}{n} = \dfrac{1}{\infty}$ OR $\dfrac{-1}{\infty}$

 even odd

$(-1)^{3004}$

$= 0$

(K) $\lim\limits_{n\to\infty} \dfrac{n/n^2}{\frac{n^2+1}{n^2}}$

$= \dfrac{0}{1+0}$

$= 0$

(l) $\lim\limits_{n\to\infty} (-1)^{n-1} n$

$= (-1)^{even} \infty$ OR $(-1)^{odd} \infty$

$= \infty = -\infty$

$\therefore$ DNE

Limits of Trigonometric Functions

y= sin *x*

y= cos *x*
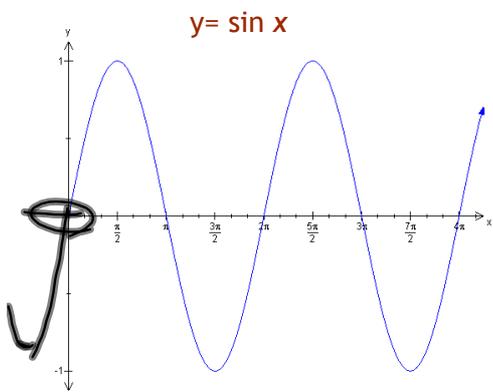
$$\lim_{x \to 0} \sin x = 0$$
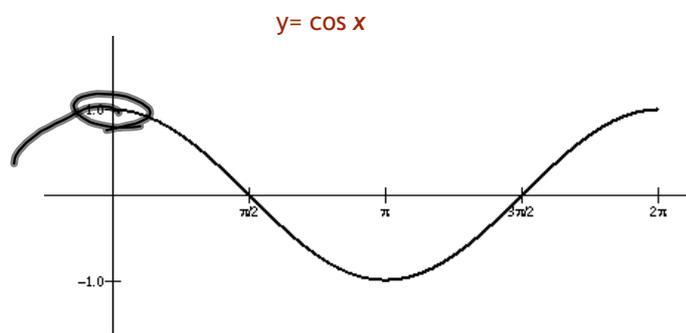
$$\lim_{x \to 0} \cos x = 1$$

The most important limit involving trigonometric functions is...

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$$

Let's examine this limit by first examining the graph of $f(\theta) = \dfrac{\sin\theta}{\theta}$
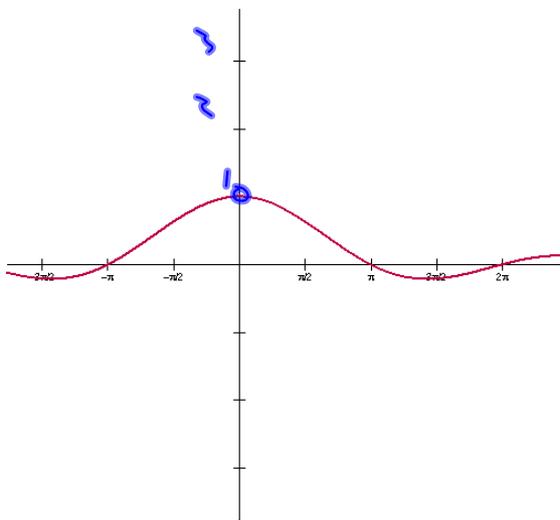
Here is a look at the table...

X	Y₁
-.03	.99985
-.02	.99993
-.01	.99998
0	ERROR
.01	.99998
.02	.99993
.03	.99985

X=-.03

IDENTITY: $\quad \lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1 \quad \text{or} \quad \lim_{\theta \to 0}\left(\frac{\sin\theta}{\theta}\right)^{n} = 1 \qquad n \in \mathbb{N}$$

Examples:

How will this identity be used?

$$\lim_{x \to 0} \frac{\sin(5x)}{x} = \frac{0}{0}$$

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$$

$$\lim_{x \to 0} \frac{8x}{\sin 5x}$$

$$\lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1$$

$$\lim_{x \to 0} 5\left(\frac{\sin 5x}{5x}\right)$$

$$= 5(1)$$

$$= 5$$

$$\lim_{x \to 0} \frac{8x}{5 \sin x}$$

$$= \frac{8}{5}(1)$$

$$\lim_{x \to 0} \left(\frac{5x}{\sin 5x}\right) \frac{8}{5}$$

$$= (1)\left(\frac{8}{5}\right)$$

$$= \frac{8}{5}$$

$$\sin 50°$$
$$\sin (5 \times 10)$$
$$5 \sin 10°$$

Pythagorean

$$\lim_{x \to \frac{\pi}{4}} \frac{4x}{\sin x}$$

$$= \frac{4\left(\frac{\pi}{4}\right)}{\sin \frac{\pi}{4}}$$

$$= \frac{\pi}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$= \pi \cdot \frac{\sqrt{2}}{1} = \boxed{\pi\sqrt{2}}$$

$$\sqrt{2} \quad 1 \quad 45° \quad 1 \quad \frac{\pi}{4}$$

$$\lim_{x \to 0} \frac{6x}{\cos 3x}$$

$$= \frac{6(0)}{\cos(0)}$$

$$= \frac{0}{1}$$

$$= 0$$

$$\lim_{x \to 0} \frac{\sin 2x}{\tan 7x}$$

Trig. Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\lim_{x \to 0} \frac{\sin 2x}{\left(\frac{\sin 7x}{\cos 7x}\right)}$$

$$\lim_{x \to 0} \sin 2x \left(\frac{\cos 7x}{\sin 7x}\right)$$

$$\lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)\left(\frac{7x}{\sin 7x}\right) \cos 7x \left(\frac{2x}{7x}\right)$$

$$= (1)(1)(1)\left(\frac{2}{7}\right)$$

$$= \frac{2}{7}$$

$$\lim_{x \to 0} \frac{3x^4}{\sin^4 2x \ (\sin 2x)^4}$$

$$\lim_{x \to 0} \left(\frac{2x}{\sin 2x}\right)^4 \frac{3}{2^4}$$

$$(1)^4 \left(\frac{3}{16}\right)$$

$$= \frac{3}{16}$$

$$\lim_{x \to 0} \frac{3\sin^5 3x}{7x^5}$$

$$\lim_{x \to 0} \frac{3^1}{7} \left(\frac{\sin 3x}{3x}\right)^5 (3)^5$$

$$= \frac{3^6}{7} (1)^5$$

$$= \frac{729}{7}$$

$$\lim_{x \to 0} \frac{\sin^3 2x}{5x^3 + 10x^4}$$

$$\lim_{x \to 0} \frac{\sin^3 2x}{5x^3(1+2x)} \longleftarrow \text{Common factor}$$

$$\lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^3 \frac{2^3}{(5)(1+2x)}$$

$$(1)^3 \left(\frac{8}{5(1+0)}\right) = \boxed{\frac{8}{5}}$$

Homework:

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#7, 9, 15, 18, 20, 22, 23, 37