Warm Up

Evaluate each of the following limits. If they do not exist provide a reason.

1. $\lim\limits_{x \to 0} \dfrac{\sin^3 2x}{\sin^3 5x}$

$$\lim_{x \to 0} \left(\frac{5x}{\sin 5x}\right)^3 \left(\frac{\sin 2x}{2x}\right)^3 \frac{8\cancel{x^3}}{125\cancel{x^3}}$$

$$= (1)^3 (1)^3 \left(\frac{8}{125}\right)$$

$$= \frac{8}{125}$$

2. $\lim\limits_{x \to 0} \dfrac{\sin^4 2x}{3x^4 - 6x^5}$

$$\lim_{x \to 0} \frac{\sin^4 2x}{3x^4(1-2x)}$$

$$\lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^4 \frac{2^4}{3(1-2x)}$$

$$= (1)^4 \left(\frac{16}{3(1-0)}\right)$$

$$= \frac{16}{3}$$

Questions ???

Practice Sheet:

#26 $\lim_{x\to 4}\left[\frac{8}{x^2-16}-\frac{1}{x-4}\right]$

$\lim_{x\to 4}\left[\frac{8}{(x-4)(x+4)}-\frac{1}{x-4}\right]$

$\lim_{x\to 4}\left[\frac{8-(x+4)}{(x-4)(x+4)}\right]$

$\lim_{x\to 4}\frac{\overset{-1}{\cancel{4-x}}}{\cancel{(x-4)}(x+4)}$

$=-\frac{1}{4+4}$

$=-\frac{1}{8}$

27/ $\lim_{x\to\infty}\frac{3^x}{4^x}$

$\lim_{x\to\infty}\left(\frac{3}{4}\right)^x$

$(<1)^\infty$

$=0$

$(<1)^\infty \Rightarrow 0$

$(>1)^\infty \Rightarrow \infty$

.

17/ $\lim_{x \to 1} \sqrt{\frac{x^3-1}{x^4-1}}$

$\lim_{x \to 1} \sqrt{\frac{(x-1)(x^2+x+1)}{(x^2-1)(x^2+1)}}$

$\lim_{x \to 1} \sqrt{\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)(x^2+1)}}$

$= \sqrt{\frac{3}{4}}$

$= \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

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#22/ $\lim_{x \to 0} \frac{\tan x}{4x}$

$\tan x = \frac{\sin x}{\cos x}$

$\lim_{x \to 0} \left(\frac{\sin x}{\cos x}\right) \cdot \frac{1}{4x}$

$\lim_{x \to 0} \left(\frac{\sin x}{x}\right) \frac{1}{4\cos x}$

$= (1)\left(\frac{1}{4(1)}\right)$

$= \frac{1}{4}$

Sheet:

#13/ $\lim_{x \to 0} \frac{x}{1-\sqrt{1-x}} \left(\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right)$

$\lim_{x \to 0} \frac{x(1+\sqrt{1-x})}{1-(1-x)}$

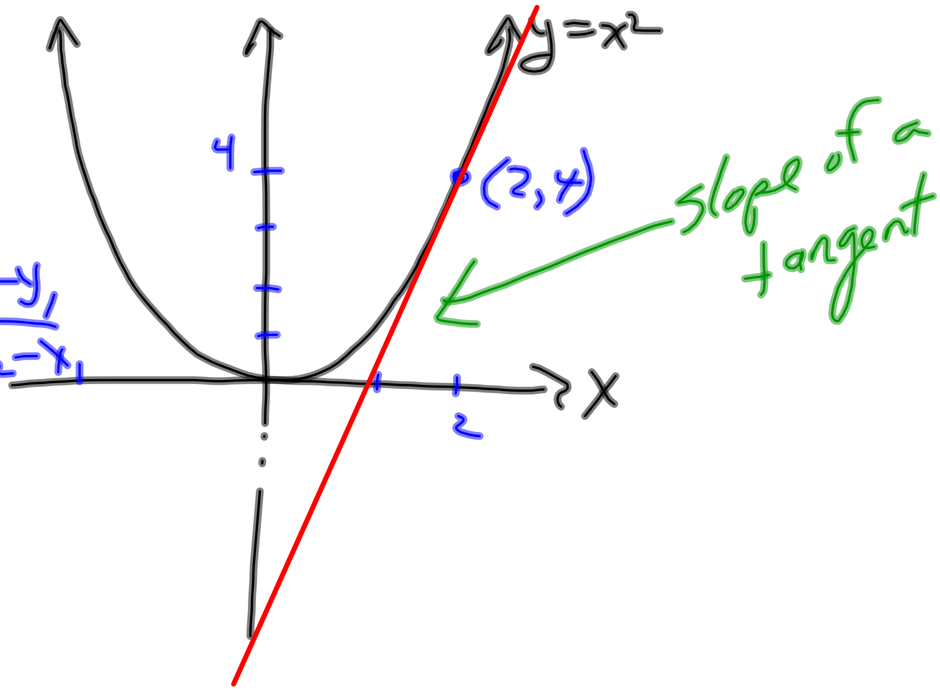
$\lim_{x \to 0} \frac{\cancel{x}(1+\sqrt{1-x})}{\cancel{x}}$

$= 1+\sqrt{1-0}$

$= 2$

Slope:

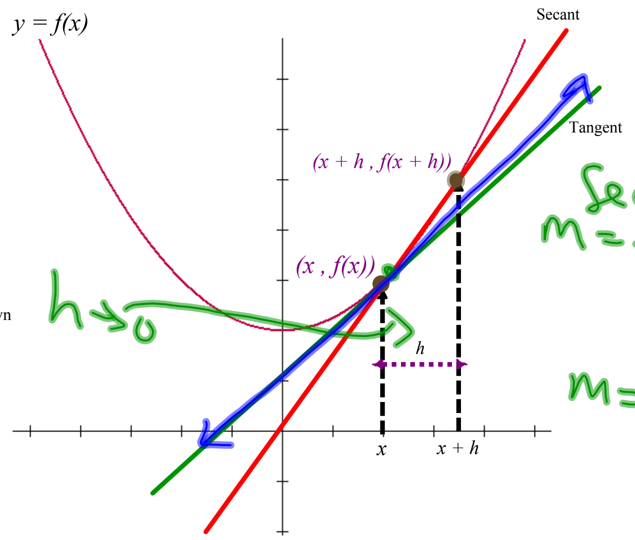
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Tangents, Velocities, and Rates of Change

Slope of a tangent to a curve:

How will we find the slope of a tangent drawn to a curve at a point x?

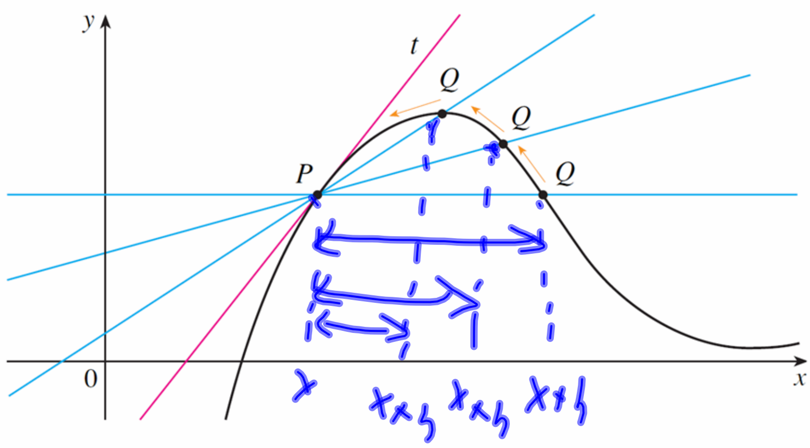


Secant . .

$$m = \frac{f(x+h) - f(x)}{x+h - x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

How will the slope of this secant become a better approximation for the slope of the tangent line?

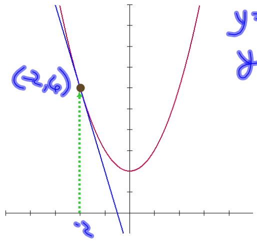


Use your knowledge of limits to determine an expression for that would represent the slope of the tangent line drawn at the point x.

$$\boxed{\text{Slope of Tangent:} \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Example:

Determine the equation of the tangent line drawn to the curve $y = x^2 + 2$ at the point $x = -2$.



$y = (-2)^2 + 2$

$y = 6$

Point-Slope:

$y - y_1 = m(x - x_1)$

Slope of a tangent:-

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Method 1: Substitute "x" at the end

$f(x) = x^2 + 2$

$f(x+h) = (x+h)^2 + 2$

$= x^2 + 2xh + h^2 + 2$

$$\lim_{h \to 0} \frac{(\cancel{x^2} + 2xh + h^2 + \cancel{2}) - (\cancel{x^2} + \cancel{2})}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$m = 2x + 0$

$m = 2x$

When $x = -2$ ... $m = 2(-2)$

$m = -4$

Method 2: Substitute "x" at beginning

$x = -2$

$$\lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h}$$

$f(-2) = (-2)^2 + 2$

$= 6$

$f(-2+h) = (-2+h)^2 + 2$

$= 4 - 4h + h^2 + 2$

$= 6 - 4h + h^2$

$$\lim_{h \to 0} \frac{(6 - 4h + h^2) - 6}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(-4 + h)}{\cancel{h}}$$

$m = -4$

Point $(-2, 6)$

$m = -4$

$y - 6 = -4(x + 2)$

$y - 6 = -4x - 8$

$(y = mx + b)$

Slope y-Intercept

$y = -4x - 2$

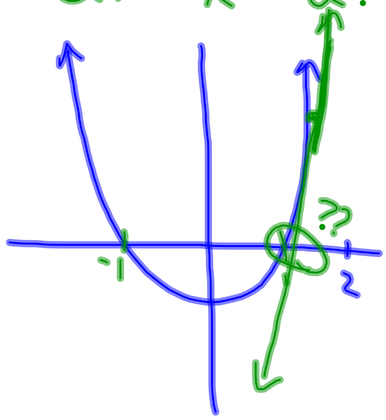
or

$Ax + By + C = 0$

Standard Form

$4x + y + 2 = 0$

Ex: Determine the x-Intercept of the tangent to $f(x)=x^2-1$ at $x=2$.



$f(x)=x^2-1$

$f(x+h)=(x+h)^2-1$

$=x^2+2xh+h^2-1$

Slope: $\lim_{h\to 0} \frac{(\cancel{x^2}+2xh+h^2-\cancel{1})-(\cancel{x^2}-\cancel{1})}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$

At $x=2$
$m=4$ ⟵ $m=2x$

$f(2)=(2)^2-1$
$=3$
$(2,3)$

$y-3=4(x-2)$

$y-3=4x-8$

$y=4x-5$

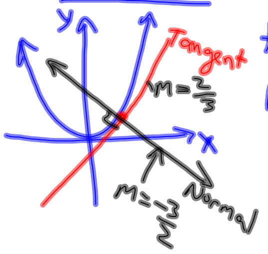
x-Int: Set $y=0$

$0=4x-5$

$\frac{5}{4}=\frac{4x}{4}$

$x=\frac{5}{4}$

NORMAL: A line drawn perpendicular to a tangent, through the point of tangency



ex. Determine the equation of the normal to the curve $y=\sqrt{2x+3}$ at $x=3$.

Slope: $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$f(x)=\sqrt{2x+3}$

$f(x+h)=\sqrt{2(x+h)+3}$

$=\sqrt{2x+2h+3}$

$$\lim_{h\to 0}\frac{\sqrt{2x+2h+3}-\sqrt{2x+3}}{h}\left(\frac{\sqrt{2x+2h+3}+\sqrt{2x+3}}{\sqrt{2x+2h+3}+\sqrt{2x+3}}\right)$$

$$\lim_{h\to 0}\frac{(2x+2h+3)-(2x+3)}{h\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}$$

$$\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}$$

$$=\frac{2}{\sqrt{2x+3}+\sqrt{2x+3}}$$

$$=\frac{\cancel{2}}{\cancel{2}\sqrt{2x+3}}$$

$$m=\frac{1}{\sqrt{2x+3}}$$

at $x=3\ldots\ m=\dfrac{1}{\sqrt{2(3)+3}}=\dfrac{1}{3}$

$\therefore$ **Normal:** $\underline{m=-3}$

$y=\sqrt{2(3)+3}$

$y=3$

$(3,3)$

$y-3=-3(x-3)$

$y-3=-3x+9$

$3x+y-12=0$

Homework:

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#7 (i), (ii), (iv) and (v)