Limits

Pg. 59

#81 $y = x^3 - 3x$

$(1, -2)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x+h)$

$= (x+h)^3 - 3(x+h)$

$= (x+h)^2(x+h) - 3(x+h)$

$= (x^2+2xh+h^2)(x+h) - 3(x+h)$

$= x^3 + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3 - 3x - 3h$

$= x^3 + 3x^2h + 3xh^2 + h^3 - 3x - 3h$

$$\lim_{h \to 0} \frac{(\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-3x} - 3h) - (\cancel{x^3} \cancel{-3x})}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3x\overset{\to 0}{h} + \overset{\to 0}{h^2} - 3)}{\cancel{h}}$$

$x = 1$

$m = 3x^2 - 3$

at $(1, -2)$

$m = 3(1)^2 - 3$

$= 0$

$y - y_1 = m(x - x_1)$

$\therefore y + 2 = 0(x-1)$

$y + 2 = 0$

$y = -2$

Pg. 61

#1 c) $\lim\limits_{x \to 1} \dfrac{\frac{1}{\sqrt{x}} - \frac{1}{1}}{x-1}$

$\lim\limits_{x \to 1} \left(\dfrac{1-\sqrt{x}}{\sqrt{x}}\right) \cdot \dfrac{1}{x-1} \left(\dfrac{1+\sqrt{x}}{1+\sqrt{x}}\right)$

$(\sqrt{x}-1)(\sqrt{x}+1)$

$\lim\limits_{x \to 1} \dfrac{\overset{-1}{\cancel{1-x}}}{\sqrt{x}\cancel{(x-1)}(1+\sqrt{x})}$

$= \dfrac{-1}{\sqrt{1}(1+\sqrt{1})}$

$= -\dfrac{1}{2}$

Pg. 59

#12 d) $\lim_{n\to\infty}\left(2-\frac{1}{n}+\frac{3}{n^2}\right)$

$= 2-\frac{1}{\infty}+\frac{3}{\infty^2}$

$= 2-0+0$

$= 2$

Practice Sheet

#1 b) $\lim_{h\to 0}\frac{(7+h)^3-7^3}{(7+h)^2-7^2}$

$$\lim_{h\to 0}\frac{\cancel{[(7+h)-7]}\,[(7+h)^2+7(7+h)+49]}{\cancel{[(7+h)-7]}\,[(7+h)+7]}$$

$$=\frac{7^2+7(7)+49}{7+7}$$

$$=\frac{147}{14}=\frac{21}{2}$$

Pg. 58

#3g) $\lim_{t\to 0} \frac{\sqrt{2+t}-\sqrt{2}}{t}\left(\frac{\sqrt{2+t}+\sqrt{2}}{\sqrt{2+t}+\sqrt{2}}\right)$

$\lim_{t\to 0} \frac{\cancel{(2+t)-2}^{\,t}}{\cancel{t}(\sqrt{2+t}+\sqrt{2})}$

$= \frac{1}{\sqrt{2}+\sqrt{2}}$

$= \frac{1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}}{4}$

Practice sheet

#1(h) $\lim_{x \to -\infty} \pi^{x}$

$= \pi^{-\infty}$

$= \dfrac{1}{\pi^{\infty}}$

$= 0$