Test:

6. x^1+2

	x^4	x^3	x^2	x^1	x^0 Rem
x^1+2	1	-3	0	2	4
		2	-10	20	-36
	1	-5	10	-18	40

$x^3-5x^2+10x-18$

7. $x-2$

$x-2$	1	-3	5	9
		-2	2	-6
	1	-1	3	15

(x^2-x+3) $R=15$

#9 ✓

Bonus:

3^{3x+5}

$$(3^5)(3^x)^3-279(3^x)^2+37(3^x)^1-1=0$$

$m=3^x$

$$243m^3-279m^2+37m-1=0$$

$m=1$, $m=\frac{1}{9}$ $\quad m=\frac{1}{27}$

$3^x=1$ $\quad 3^x=\frac{1}{9}$ $\quad 3^x=\frac{1}{27}$

$x=0$

$x=-2$ $\quad x=-3$

1\. a) $(x-2)(x+4)(3x-2)$

b) $3(2x^2-y)(2x^2+y)(16x^8+4x^4y^2+y^4)$

2\. $Q: (6x^2-x+18)$ $Rem = 50$

$$\begin{array}{r} -x^3 \\ -18x^2 \\ \hline -x^3 \end{array}$$

3\. $k=-3$, $p=-3$

$$\begin{array}{r} 2k-3p=3 \\ -8k+3p=15 \\ \hline -6k=18 \\ k=-3 \end{array}$$

4\. $x=-4, -\frac{1}{3}, \frac{2}{5}$

5\. $625x^{12}-2000x^9y^2+2400x^6y^4-1280x^3y^6+256y^8$

6\. $\underset{3003}{{}_{14}C_8}\,\underset{(64)}{(2x^3)^6}\,\underset{(6561)}{(3y^5)^8}$

$= 1\ 260\ 971\ 712$

7\. $x=\frac{2}{3}$, $x=-1\pm2\sqrt{5}$

$3x-2=0$

$x=-1\pm2\sqrt{5}$

$(x+1)^2=(\pm2\sqrt{5})^2$

$x^2+2x+1=20$

$x^2+2x-19=0$

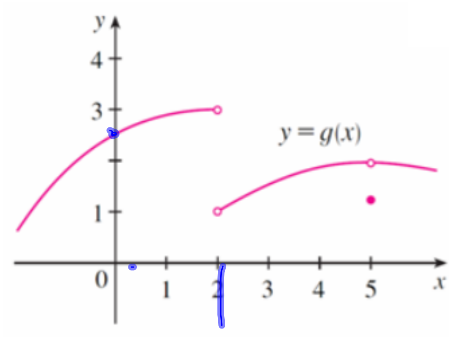
$(3x-2)(x^2+2x-19)=0$

$3x^3+4x^2-61x+38=0$

One-sided limits

Use the graph shown below to evaluate the following limits:

$\lim_{x\to 0} f(x)$



$\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$

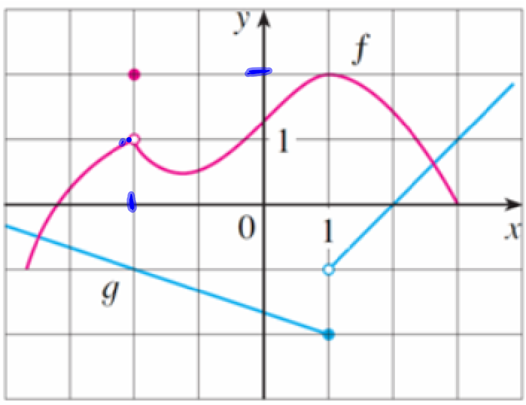
1. $\lim_{x\to 2^-} g(x) = \boxed{3}$ 2. $\lim_{x\to 2^+} g(x) = \boxed{1}$ 3. $\lim_{x\to 2} g(x) = \boxed{\text{DNE}}$

"as x approaches 2 from the left" "as x approaches 2 from the right"

4. $\lim_{x\to 5^-} g(x) = \boxed{2}$ 5. $\lim_{x\to 5^+} g(x) = \boxed{2}$ 6. $\lim_{x\to 5} g(x) = \boxed{2}$

Notice... $g(5) = 1$

Example:



Evaluate each of the following:

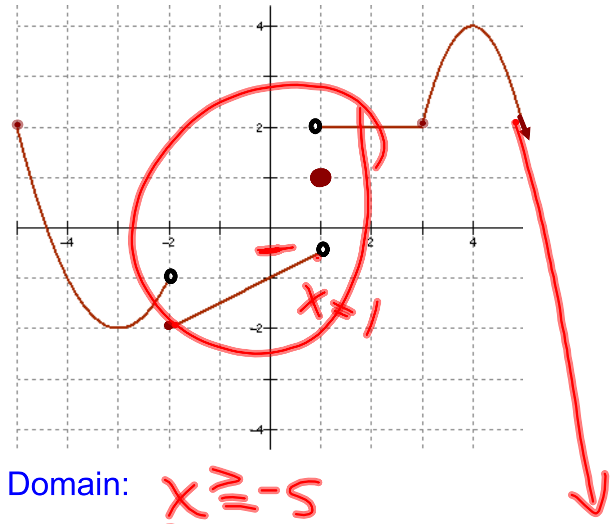
$f(-2) = 2$ $\lim_{x\to 1^-} g(x) = -2$ $g(1) = -2$

$\lim_{x\to 1^+} g(x) = -1$ $\lim_{x\to 1} g(x) = \text{DNE}$ $\lim_{x\to 1} f(x) = 2$

$\lim_{x\to -2} f(x) = 1$

EXAMPLE...

Evaluate the following using the graph of $f(x)$ shown below...



Domain: $x \geqq -5$

Range: $[-5, \infty)$

$y \leqq 4$

$(-\infty, 4]$

1. $\lim\limits_{x \to -2^-} f(x) = -1$

2. $\lim\limits_{x \to -2^+} f(x) = -2$

3. $\lim\limits_{x \to -2} f(x) =$ DNE

4. $f(-2) = -2$

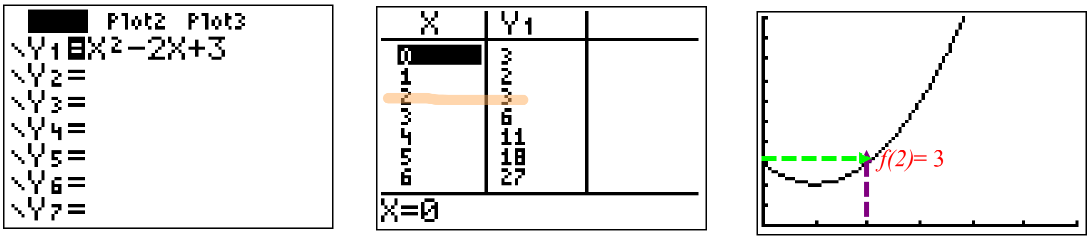
5. $f(1) = 1$

6. $\lim\limits_{x \to 1^-} f(x) = -\frac{1}{2}$

7. $\lim\limits_{x \to 1^+} f(x) = 2$

Limit of a Function

Let's examine the function $f(x) = x^2 - 2x + 3$



We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.

X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

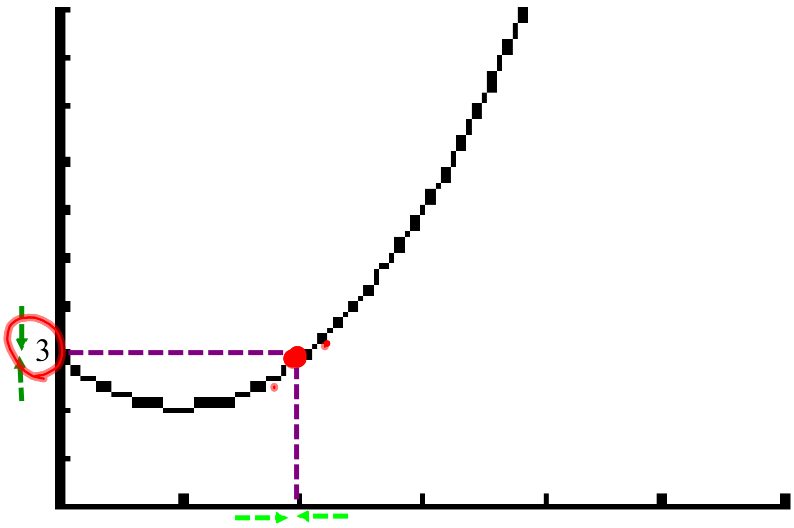
X=1.85

As x gets closer to 2 from the left y is getting closer to 3.

As x gets closer to 2 from the right y is getting closer to 3.

From the above, the notion of the limit of a function arises...

$f(x)$ approaches 3



As x approaches 2

Notation: $\lim_{x \to 2} f(x) = 3$

"The limit of the function $f(x)$ as x approaches 2 is equal to 3."

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2}\frac{x^2-2x+1}{x+3}$$

$$=\frac{(-2)^2-2(-2)+1}{-2+3}$$

$$=\frac{9}{1}$$

$$=9$$

$$\lim_{x\to 3}(16-x^2)$$

$$=16-(3)^2$$

$$=7$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

$\Rightarrow$ Factor
$\Rightarrow$ Rationalize
$\Rightarrow$ Expand
$\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x\to 3}\frac{x^2-6x+9}{9-x^2}=\frac{9-18+9}{9-9}=\frac{0}{0}$$

$$\lim_{x\to 3}\frac{(x-3)\cancel{+}(-1)}{\cancel{(3-x)}(3+x)}$$

$$=\frac{(3-3)(-1)}{(3+3)}$$

$$=\frac{0}{6}=0$$

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$$

$$\lim_{h\to 0}\frac{(4+h)-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{\cancel{h}\,1}{\cancel{h}(\sqrt{4+h}+2)}$$

$$=\frac{1}{\sqrt{4+0}+2}=\frac{1}{4}$$