3. $f(x)=\frac{7\sqrt{x}}{4}-\frac{6}{x^5}+\frac{10x^3}{\sqrt[4]{x}}+6x^{30}-ex+9$ $\qquad \frac{x^3}{x^{1/4}}$

$\frac{x^{12/4}}{x^{1/4}}$

$f(x)=\frac{7}{4}x^{1/2}-6x^{-5}+10x^{\frac{11}{4}}+6x^{30}-ex+9$

$f'(x)=\frac{7}{8}x^{-1/2}+30x^{-6}+\frac{55}{2}x^{7/4}+180x^{29}-e$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

1st 2nd $f(x)g'(x)+g(x)f'(x)$

Express the product rule verbally if you are considering a function of the form...

$f(x)$= (First) x (Second)

" The derivative of the product of two functions is the derivative of the first multiplied by the second, plus the first multiplied by the derivative of the second"

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$y=6x^2$

$y'=12x$

$y=6x^2$

$y'=\cancel{0(x^2)}+6(2x)$

$=12x$

Examples:

$$f(x) = \underbrace{(7x^3 - x^2 + 5)}_{1^{st}}\underbrace{(x^9 + 3x - 5)}_{2^{nd}}$$

DO NOT SIMPLIFY!!

$$F'(x) = (21x^2 - 2x)(x^9 + 3x - 5) + (7x^3 - x^2 + 5)(9x^8 + 3)$$

$$h(t) = (t^3 - 5t)(6\sqrt{t} - t^{-5})$$

$$h'(t) = (3t^2 - 5)(6\sqrt{t} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

$$g(x) = \underbrace{(7x^3 - 5)(4x^2 - 2x + 3)}_{1^{st}}\underbrace{(9 - x^6)}_{2^{nd}}$$

$$g'(x) = \left[(21x^2)(4x^2 - 2x + 3) + (7x^3 - 5)(8x - 2)\right](9 - x^6) + \left[(7x^3 - 5)(4x^2 - 2x + 3)\right](-6x^5)$$

$$f(x) = (x^{12} - 5x^3)(7x^3 + x - 5)(4\sqrt{x} + 2)$$

$$F'(x) = \left[(12x^{11} - 15x^2)(7x^3 + x - 5) + (x^{12} - 5x^3)(21x^2 + 1)\right](4\sqrt{x} + 2) + \left[(x^{12} - 5x^3)(7x^3 + x - 5)\right](2x^{-1/2})$$

$$h(x) = \underbrace{f(x)\,g(x)}_{1^{st}}\,\underbrace{r(x)}_{2^{nd}}$$

$$h'(x) = \left[f'(x)\,g(x) + f(x)\,g'(x)\right] r(x) + f(x)\,g(x)\,r'(x)$$

$$h'(x) = f'(x)g(x)r(x) + g'(x)f(x)r(x) + r'(x)f(x)g(x)$$

$$f(x) = (x^2-3)(x^3+7x)(8x^{-2}-x)(15x^2+3)$$

$$f'(x) = (2x)(x^3+7x)(8x^{-2}-x)(15x^2+3) + (3x^2+7)(x^2-3)(8x^{-2}-x)(15x^2+3)$$

$$+ (-16x^{-3}-1)(x^2-3)(x^3+7x)(15x^2+3) + (30x)(x^2-3)(x^3+7x)(8x^{-2}-x)$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\overset{f'(x)}{\frac{d}{dx}}[f(x)] - f(x)\overset{g'(x)}{\frac{d}{dx}}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally ...

" The derivative of the numerator multiplied by the denominator, minus the numerator multiplied by the derivative of the denominator all over the denominator squared"

Examples:

$$f(x) = \frac{x^3 - 7x^2 + 2}{x^8 - 4x^5}$$

$$f'(x) = \frac{(3x^2 - 14x)(x^8 - 4x^5) - (x^3 - 7x^2 + 2)(8x^7 - 20x^4)}{(x^8 - 4x^5)^2}$$

$$f(x)=\frac{8-9x^7}{3x-7}$$

$$f'(x)=\frac{(-63x^6)(3x-7)-(8-9x^7)(3)}{(3x-7)^2}$$

$$f(x)=\frac{(10x^{-5}+x)(3x^3+5)}{\left(-2x^6+\sqrt[3]{x}\right)}$$

$$f'(x)=\frac{\left[(-50x^{-6}+1)(3x^3+5)+(10x^{-5}+x)(9x^2)\right](-2x^6+x^{1/3})-\left[(10x^{-5}+x)(3x^3+5)\right]\left(-12x^5+\frac{1}{3}x^{-2/3}\right)}{\left(-2x^6+\sqrt[3]{x}\right)^2}$$

$$f(x)=\frac{(x-7)(2x^6-x^4+5)}{(6x-x^5)(4x^3+2)}$$

$$f'(x)=\frac{\left[(1)(2x^6-x^4+5)+(x-7)(12x^5-4x^3)\right]\left[(6x-x^5)(4x^3+2)\right]-(x-7)(2x^6-x^4+5)\left[(6-5x^4)(4x^3+2)+(6x-x^5)(12x^2)\right]}{\left[(6x-x^5)(4x^3+2)\right]^2}$$

Chain Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$\cancel{F'(x) = f'(g(x))g'(x)} \quad f(x) = [g(x)]^n, n \in \mathbb{R}$$

$$f'(x) = n[g(x)]^{n-1} \cdot g'(x)$$

Examples:

$$f(x) = (5x^3+1)^{10}$$

$$F'(x) = 10(5x^3+1)^9(15x^2)$$

$$h(x) = \sqrt[3]{5-3x^4} = (5-3x^4)^{1/3}$$

$$h'(x) = \frac{1}{3}(5-3x^4)^{-2/3}(-12x^3)$$

$$(9x)^{-3}$$

$$g(x) = \underbrace{9x^{-3}}\underbrace{(5x^3-1)^6}$$

$$g'(x) = (-27x^{-4})(5x^3-1)^6 + 9x^{-3}\left[6(5x^3-1)^5(15x^2)\right]$$

$$g(x) = \frac{(x^2-5x+1)^8}{(1-x^{-7})^{20}}$$