Warm Up

Evaluate the following limits, if they exist:

1. $\lim_{x \to 8} \dfrac{x-8}{\sqrt[3]{x}-2}$ 12

2. $\lim_{x \to 3} \dfrac{x^{-2}-3^{-2}}{x-3}$ $\frac{-2}{27}$

3. $\lim_{x \to 1} \dfrac{x^3-1}{x^3-x^2-4x+4}$ $=-1$

4. $\lim_{h \to 0} \dfrac{(a+h)^2-a^2}{h}$ $=2a$

① Clue: $x-10$

Factor: $(\sqrt{x}-\sqrt{10})(\sqrt{x}+\sqrt{10})$

OR

$(\sqrt[3]{x}-\sqrt[3]{10})((\sqrt[3]{x})^2+\sqrt[3]{10}\sqrt[3]{x}+\sqrt[3]{10}^2)$

1. $\lim_{x \to 8} \frac{x-8}{\sqrt[3]{x}-2}$

Diff. of cubes

$$\lim_{x \to 8} \frac{\cancel{(\sqrt[3]{x}-2)}(\sqrt[3]{x}^2+2\sqrt[3]{x}+4)}{\cancel{(\sqrt[3]{x}-2)}}$$

$$= (\sqrt[3]{8})^2+2\sqrt[3]{8}+4$$

$$= 4+4+4$$

$$= 12$$

$\lim_{x \to 3} \frac{x^{-2}-3^{-2}}{x-3}$

$$\lim_{x \to 3} \frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$$

$$\lim_{x \to 3} \left(\frac{9-x^2}{9x^2}\right) \cdot \frac{1}{x-3}$$

$$\lim_{x \to 3} \frac{\overset{-1}{\cancel{(3-x)}}(3+x)}{9x^2}\left(\frac{1}{\cancel{x-3}}\right)$$

$$= \frac{-1(6)}{9(9)}$$

$$= \frac{-6}{81} = \frac{-2}{27}$$

3. $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x^3 - x^2 - 4x + 4}$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+2)(x-2)}$$

$$= \frac{3}{(3)(-1)}$$

$$= -1$$

$x = 1$

-1	1	-1	-4	4
		-1	0	4
	1	0	-4	0

$(x-1)(x^2-4)$

$(x-1)(x-2)(x+2)$

4. $\lim\limits_{h \to 0} \dfrac{(a+h)^2 - a^2}{h}$

$$\lim_{h \to 0} \frac{\cancel{[(a+h)-a]}^{\cancel{h}}\,[(a+h)+a]}{\cancel{h}}$$

$$= a + 0 + a$$

$$= 2a$$

Pg. 20

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b) $$\lim_{x \to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}\left(\frac{\sqrt{6-x}+2}{\sqrt{6-x}+2}\right)\left(\frac{\sqrt{3-x}+1}{\sqrt{3-x}+1}\right)$$

Rationalize Both

$$\lim_{x \to 2} \frac{[(6-x)-4](\sqrt{3-x}+1)}{[(3-x)-1](\sqrt{6-x}+2)}$$

$$\lim_{x \to 2} \frac{\cancel{(2-x)}(\sqrt{3-x}+1)}{\cancel{(2-x)}(\sqrt{6-x}+2)}$$

$$= \frac{\sqrt{3-2}+1}{\sqrt{6-2}+2}$$

$$= \frac{2}{4} = \frac{1}{2}$$

6.h) $\lim_{x \to 4} \dfrac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4}$

$\lim_{x \to 4} \left(\dfrac{2-\sqrt{x}}{2\sqrt{x}}\right) \cdot \dfrac{1}{x-4} \left(\dfrac{2+\sqrt{x}}{2+\sqrt{x}}\right)$ Rationalize

OR factor

$(\sqrt{x}-2)(\sqrt{x}+2)$

$\lim_{x \to 4} \dfrac{\cancel{4-x}^{\,-1}}{2\sqrt{x}\,\cancel{(x-4)}\,(2+\sqrt{x})}$

$= \dfrac{-1}{(2\sqrt{4})(2+\sqrt{4})}$

$= -\dfrac{1}{16}$

6.J) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-x}\left(\frac{\sqrt{x}+x}{\sqrt{x}+x}\right)$

$$\lim_{x\to 1} \frac{(x-1)(\sqrt{x}+x)}{x-x^2}$$

$$\lim_{x\to 1} \frac{\overset{-1}{\cancel{(x-1)}}(\sqrt{x}+x)}{x\cancel{(1-x)}}$$

$$= \frac{-1(1+1)}{1}$$

$$= -2$$

5. a) $\lim_{h\to 0} \frac{(4+h)^3-64}{h}$

$\lim_{h\to 0} \frac{\left[\cancel{(4+h)-4}\right]\left[(4+h)^2+4(4+h)+16\right]}{\cancel{h}}$

$= 4^2+4(4)+16$

$= \underline{48}$

5.f) $\lim\limits_{h \to 0} \dfrac{\dfrac{1}{(2+h)^2} - \dfrac{1}{4}}{h}$

$$\lim_{h \to 0} \left[\frac{4-(2+h)^2}{4(2+h)^2} \right] \cdot \frac{1}{h}$$

$$\lim_{h \to 0} \left[\frac{\overset{-h}{\cancel{[2-(2+h)]}}\,[2+(2+h)]}{4(2+h)^2} \right] \frac{1}{\cancel{h}}$$

$$= \frac{-1(2+2)}{4(2+0)^2}$$

$$= -\frac{4}{16}$$

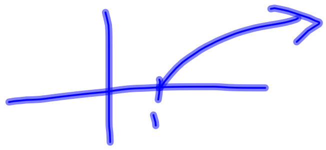
$$= -\frac{1}{4}$$

Recall from our prior discussions that ...

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Let's look at a couple of unique functions:

1) $\lim_{x \to 1} \sqrt{x-1}$



$\lim_{x \to 1^+} \sqrt{1.000..1 - 1}$

$\sqrt{0^+}$

$= 0$

$\lim_{x \to 1^-} \sqrt{0.99\bar{9} - 1}$

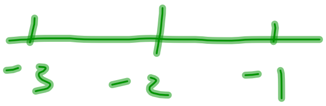
$\sqrt{-0.000..1}$

DNE

$\therefore$ DNE

* A single Even indexed Radical that is approaching "0"

2) $\lim_{x \to -2} \frac{|x+2|}{x+2} = \text{DNE}$



$\lim_{x \to -2^+} \frac{|-1.99... + 2|}{-1.999.. + 2}$

$\frac{\text{small } (+) \#}{\text{same small } (+) \#}$

$= 1$

$\lim_{x \to -2^-} \frac{|-2.000..1 + 2|}{-2.000..1 + 2}$

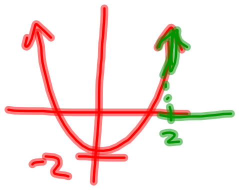
$= \frac{\cancel{\text{small } (+) \#}}{\cancel{\text{same small } \#} (-)}$

$= -1$

Piecewise Defined Functions

Definition:
- **Functions defined by different formulas in different parts of their domains**

Example:



$$f(x) = \begin{cases} x+3 & if \ x \le 2 \\ x^2 - 2 & if \ x > 2 \end{cases}$$

1) Determine *f(1)* , *f(3),* and *f(2)*.
2) Sketch *f(x)*.

$f(1) = 4$ ($x<2$) $= 1+3$

$f(3) = 7$ ($x>2$) $= 3^2 - 2$

$f(2) = 5$ $= 2+3$

$y = x+3 \quad x \le 2$

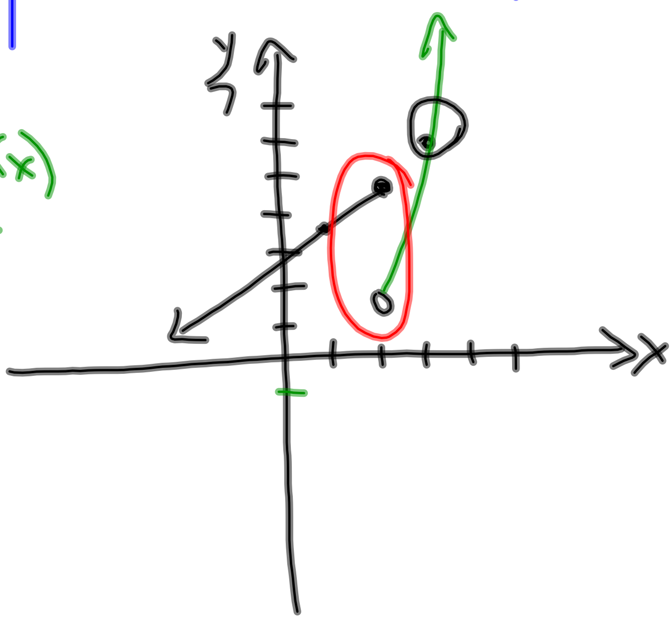
x	y
2	5
1	4

$y = x^2 - 2, \; x > 2$

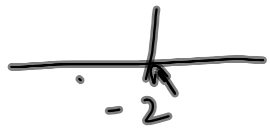
x	y
2	2
3	7

V(0,-2) opens up } Parabola

$\lim_{x \to 2} f(x) = DNE$



$$f(x)=\begin{cases}\frac{1}{2}x-2 & if\ \ x<-2\\ -1 & if\ -2\le x\le 1\\ (x-2)^2+1 & if\ \ x>1\end{cases}$$



$\lim_{x\to -2^-} f(x) \leftarrow x<-2$

$=\frac{1}{2}(-2)-2$

$=-3$

$\lim_{x\to -2^+} f(x) = -1$

Sketch the following piecewise function: