Warm-Up

Simplify:

$-32a^{12}b^{9}$

BEDMAS

$$\frac{\left(4^{1}a^{2}b^{5}\right)^{3}\left(-2a^{4}b\right)^{3}}{-8a^{5}b^{3}\cdot\left(-2a^{1}b^{6}\right)}$$

$$\frac{\left(64a^{6}b^{15}\right)\left(-8a^{12}b^{3}\right)}{16a^{6}b^{9}}$$

$$\frac{-512a^{18}b^{18}}{16a^{6}b^{9}} \div$$

$$\boxed{= -32a^{12}b^{9}}$$

Simplify:

$$= {}^{-}9x^{11}y^{8}$$

$$\frac{(-2x^4y^7)^3(-3x^5)^2}{(2x^3y)(-2x^4y^6)^2} \qquad (-3x^5)^2 = (-3x^5)(-3x^5)$$

$$= \frac{(-8x^{12}y^{21})(9x^{10})}{(2x^3y^1)(4x^8y^{12})} = \frac{-72x^{22}y^{21}}{8x^{11}y^{13}}$$

$$= {}^{-}9x^{11}y^{8}$$

1 b) $(0.35)^5 = 0.00525\ldots$

$= 0.01$ $\quad$ $5.25\ 10^{-3}$

4. f) $2\sqrt[4]{64}$ $\quad$ $2^4 = 16$

$2\left(\sqrt[4]{16} \cdot \sqrt[4]{4}\right)$

$2\left(2\sqrt[4]{4}\right)$

$4\sqrt[4]{4}$

5. a) $\sqrt{80}$

b) $-10\sqrt{6}$

$-\sqrt{10^2 \cdot 6}$

$-\sqrt{600}$

c) $5\sqrt[4]{3}$

index $\sqrt[4]{5^4 \cdot 3}$

$\sqrt[4]{1875}$

$-(3 \times 3)$

$-3^2 \Rightarrow -9$

$(-3)^2$

4.4 Fractional Exponents and Radicals

LESSON FOCUS	Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?

$0.87^{\frac{1}{2}} \Rightarrow 1 \; \{ \; 0.87$

Recall from past work with exponents...

Make Connections

Recall the exponent laws for integer bases and whole number exponents.

Product of powers: $a^m \cdot a^n = a^{m+n}$

*Quotient of powers: $a^m \div a^n = a^{m-n}, a \neq 0$

Power of a power: $(a^m)^n = a^{mn}$

Power of a product: $(ab)^m = a^m b^m$

Power of a quotient: $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, b \neq 0$

How about zero as an exponent?

$b^0 = \underline{\underline{1}}$

$\dfrac{3^7}{3^7} = 3^0 = 1$

Connecting Radicals and Exponents:

Time to continue our development of the properties of radicals...

What is the value of each of the following:

$\sqrt{5} = 5^{1/2}$

$$\sqrt{5} \bullet \sqrt{5} = \sqrt{25}$$

$$5^{?} \cdot 5^{?} = 5^{1}$$

$$\sqrt[3]{3} \times \sqrt[3]{3} \times \sqrt[3]{3} = \sqrt[3]{27}$$

$$3^{1/3} \cdot 3^{1/3} \cdot 3^{1/3} = 3^{1}$$

How about the following:

$$x^{\frac{1}{2}} \bullet x^{\frac{1}{2}} = x^{1}$$

$$\pi^{\frac{1}{3}} \bullet \pi^{\frac{1}{3}} \bullet \pi^{\frac{1}{3}} = \pi$$

Based on the previous slide it would seem that...

$$\sqrt{x} = x^{\frac{1}{2}} \qquad \sqrt[3]{x} = x^{\frac{1}{3}}$$

Generally then... $\sqrt[n]{x} = x^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is a natural number and x is a rational number, $x^{\frac{1}{n}} = \sqrt[n]{x}$

Denominator will be the index of radical form

$9^{1/2} = \sqrt{9}$
$= 3$

$27^{1/3} = \sqrt[3]{27}$
$= 3$

$16^{1/4} = \sqrt[4]{16}$
$= 2$

$125^{1/3} = \sqrt[3]{125}$
$= 5$

What about when numerator in NOT a 1??

ie. Something like $8^{\frac{5}{3}}$??

Let's relate back to exponent laws...which one would help?

$\frac{1}{3} \cdot \frac{5}{1} = \frac{5}{3}$

Exponent of Radical

$$\left(8^{\frac{1}{3}}\right)^5 = \left(\sqrt[3]{8}\right)^5 = 32$$

$$16^{3/4} = \left(16^{1/4}\right)^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$$

Index

$$8^{\frac{5}{3}} = \left(8^5\right)^{1/3} = \sqrt[3]{8^5}$$