Warm Up:

1. Evaluate each of the following without using a calculator:

$(a)\ 27^{\frac{1}{3}} = ?$ $\quad \sqrt[3]{27}$

$= 3$

$(b)\ 64^{\frac{3}{2}} = ?$ $\quad (\sqrt{64})^3$

$= 8^3$

$8 \times 8 \times 8$

64×8

$= 512$

$(c)\left(\frac{16}{81}\right)^{\frac{3}{4}} = ?$ $\quad \sqrt[3]{64}$

8

512

$\frac{16^{3/4}}{81^{3/4}} = \frac{(\sqrt[4]{16})^3}{(\sqrt[4]{81})^3}$

$= \frac{8}{27}$

$(d)\ 32^{\frac{7}{5}} = ?$ $\quad (\sqrt[5]{32})^7$

$= 2^7$

$= 128$

2. Evaluate each of the following using a calculator:

$(a) -32^{\frac{2}{7}} = ?$

$\approx -2.6918\ldots$

$(b)\ 20^{\frac{5}{8}} = ?$

$\approx 6.5034\ldots$

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#20/ $100(0.5)^{\frac{n}{5}}$

a) $\frac{1}{2}h \Rightarrow n = 0.5$

$100(0.5)^{\frac{0.5}{5}}$

$= 93.30$

$= 93\%$

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$\frac{1}{2}$ hour later ...

$100(0.87)^{\frac{1}{2}}$

$= 93.27..$

$= 93\%$

b) $n = 1.5$

$100(0.5)^{1.5/5}$ $100(0.5)\wedge(1.5/5)$

81.22%

c) $\frac{100(0.5)^{\frac{n}{5}}}{100} = \frac{50}{100}$

$(0.5)^{n/5} = \frac{1}{2}$

$\left(\frac{1}{2}\right)^{n/5} = \left(\frac{1}{2}\right)^{1}$

$\frac{n}{5} = 1$ (5)

$n = 5$

$\frac{3w}{3} = \frac{15}{3}$

$w = 5$

$6^{2} = 6^{w}$

$w = 2$

21. $T \doteq 0.2R^{3/2}$

Has to be larger

Earth:

$T = 0.2(149)^{3/2}$

$T =$

Mars:

$T = 0.2(228)^{3/2}$

$T =$

4.5 Negative Exponents and Reciprocals

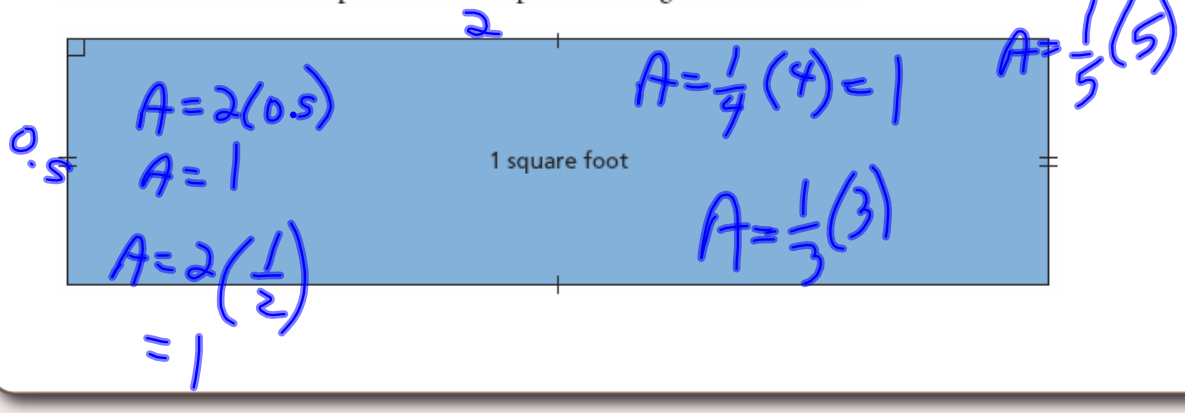
LESSON FOCUS Relate negative exponents to reciprocals.

Reciprocal

$-\frac{3}{5} \Rightarrow -\frac{5}{3}$

Make Connections

A rectangle has area 1 square foot.
List 5 possible pairs of lengths and widths for this rectangle.
What is the relationship between the possible lengths and widths?



Reciprocals:

What are some properties of numbers classified as reciprocals?

Product of a number and it's reciprocal will ALWAYS equal 1.

ie $-\frac{3}{7} \cdot -\frac{7}{3} = \frac{21}{21} = 1$

Definition:

Two numbers with a product of 1 are reciprocals.
Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.
Similarly, $\frac{2}{3} \cdot \frac{3}{2} = 1$, so the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are also reciprocals.

Use the concept of reciprocals to deal with NEGATIVE exponents...

We define powers with negative exponents so that previously developed properties such as $a^m \cdot a^n = a^{m+n}$ and $a^0 = 1$ still apply.

How can we explain the meaning of negative exponents?

$5^3 = 125$ $\qquad$ $5^{-3} = ?$

$$\frac{5^7}{5^{10}} = 5^{-3}$$

$$\frac{1}{\cancel{5\times5\times5\times5\times5\times5\times5}}{\cancel{5\times5\times5\times5\times5\times5\times5}\times5\times5\times5}$$

$$\frac{1}{5\times5\times5}$$

$$\frac{1}{5^3} = 5^{-3}$$

Take Reciprocal and make exponent positive

$$\frac{1}{125} = 5^{-3}$$

IMPORTANT PROPERTY!!

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Examples:

$6^{-2} = \frac{1}{6^2}$

$= \frac{1}{36}$

$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$

$= \frac{125}{8}$

$\frac{3^{-2}}{4} = \frac{1}{4(3)^2}$

$= \frac{1}{36}$

OR??

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2}$

$= \frac{16}{9}$

OR ...

$\frac{3}{4^{-2}} = 3(4)^2$

$= 3(16)$

$= \frac{48}{1}$

$0.2^{-4} = \left(\frac{1}{5}\right)^{-4}$

$= \left(\frac{5}{1}\right)^{4}$

$= 625$

ex.

1) -5^{-2}

$= -\frac{1}{5^2}$

$= -\frac{1}{25}$

2) $(-6)^{-2}$

$= \frac{1}{(-6)^2} = \frac{1}{36}$

3) $\left(\frac{3}{4}\right)^{-3}$

$\left(\frac{4}{3}\right)^3 = \frac{64}{27}$

4) $\frac{7}{(2^{-3})}$

$7(2^3)$

$= 56$

5) $\frac{2^{-3}}{9}$

$\frac{1}{2^3(9)}$

$\frac{1}{72}$.

6) $-\frac{3^{-1}}{8}$

$-\frac{1}{3^1(8)}$

$= -\frac{1}{24}$