4.6 Applying the Exponent Laws

THINK ABOUT IT

Work on your own.

What is the value of $\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$ when $a = -3$ and $b = 2$?

$\left(\frac{(-3)^6(2)^9}{(-3)^5(2)^8}\right)^{-2}$

$(ab)^{-2}$

Simplify first

$((-3)(2))^{-2}$

$(-6)^{-2}$

$= \frac{1}{(-6)^2} = \frac{1}{36}$

THINK ABOUT IT

Work on your own.

What is the value of $\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$ when $a = -3$ and $b = 2$?

$\left(\frac{a^5b^8}{a^6b^9}\right)^2 = \frac{a^{10}b^{16}}{a^{12}b^{18}} = a^{-2}b^{-2}$

$= \frac{1}{a^2b^2} = \frac{1}{(-3)^2(2)^2} = \frac{1}{9(4)}$

$= \frac{1}{36}$

Let's put all of our exponent skills to the test...

Don't forget the basic laws:

Make Connections

Recall the exponent laws for integer bases and whole number exponents.

Product of powers:	$a^m \cdot a^n = a^{m+n}$
Quotient of powers:	$a^m \div a^n = a^{m-n}, a \neq 0$
Power of a power:	$(a^m)^n = a^{mn}$
Power of a product:	$(ab)^m = a^m b^m$
Power of a quotient:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Warm Up

Simplify or evaluate each of the following:

1. $(-3)^2 =$ 9

2. $-3^2 =$ -9

$(3+4)^2 \neq 3^2+4^2$

$=25$

3. $(2x^3y^6)^4 =$ $16x^{12}y^{24}$

4. $\dfrac{(-5a^3)(2a^2)^3}{(2a^3)^2} =$ $\dfrac{(-5a^3)(8a^6)}{4a^6}$

$=\dfrac{-40a^9}{4a^6}$

$=-10a^3$

5. $4w^0 =$ $4(1)$ $=4$

6. $5^{-2} =$ $\dfrac{1}{5^2}=\dfrac{1}{25}$

7. $\dfrac{2^{-1}}{3} =$ $\dfrac{1}{3(2)^1}$ $=\dfrac{1}{6}$

8. $(2^3-3^2)^{10} =$ $(8-9)^{10}$ $(-1)^{10}=1$

Power of a Sum does Not exist!!

9. $5^8 \times (5^3)^{12} \div 5^8 \times (5^7)^2 =$

$5^8 \times 5^{36} \div 5^8 \times 5^{14}$

$5^{44} \div 5^8 \times 5^{14}$

$5^{36} \times 5^{14}$

$=5^{50}$

$(3+5)^3$ NOT (3^3+5^3)

$= (8)^3$

$= 512$

$= 27+125$

$= 152$ ~~152~~

Fractions:

$\frac{3}{4} + \frac{2}{5}$

$\frac{15}{20} + \frac{8}{20}$

$= \frac{23}{20}$

$1\frac{3}{20}$

$\frac{7}{3} - \frac{1}{2} + \frac{3}{4}$

$\frac{28 - 6 + 9}{12}$

$= \frac{31}{12} = 2\frac{7}{12}$

$\frac{0}{1} + \frac{2}{5}$

$\frac{6}{1} - \frac{3}{7}$

Example 3 Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTION

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)}$ Using the power of a power law.

$= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2$

$= 2ab^2$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) = x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1}$ Use the product of powers law.

$= x^{\frac{3}{2}+\frac{1}{2}} \cdot y^{2+(-1)}$

$= x^2y$

(Solution continues.)

Example 3 Simplifying Algebraic Expressions with Rational Exponents

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}} = \dfrac{4}{2} \cdot \dfrac{a^{-2}}{a^{2}} \cdot \dfrac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}}$ Use the quotient of powers law.

$= 2 \cdot a^{(-2)-2} \cdot b^{\frac{2}{3}-\frac{1}{3}}$

$= 2 \cdot a^{-4} \cdot b^{\frac{1}{3}}$ Write with a positive exponent.

$= \dfrac{2b^{\frac{1}{3}}}{a^{4}}$

(Solution continues.)

Example 3 Simplifying Algebraic Expressions with Rational Exponents

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(\dfrac{100}{25}\cdot\dfrac{a^1}{a^5}\cdot\dfrac{1}{b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$ Simplify inside the brackets first. Use the quotient of powers law. Write with a positive exponent.

$= \left(4\cdot a^{1-5}\cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= \left(4\cdot a^{-4}\cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$ Use the power of a power law.

$= 4^{\frac{1}{2}}\cdot a^{(-4)\left(\frac{1}{2}\right)}\cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$

$= 2\cdot a^{-2}\cdot b^{\frac{1}{4}}$ Write with a positive exponent.

$= \dfrac{2b^{\frac{1}{4}}}{a^2}$

CHECK YOUR UNDERSTANDING

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$ $\frac{3}{1}\cdot\frac{1}{3}$ $=\frac{3}{3}$ $=1$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$ $\frac{6}{1}\times\frac{1}{3}=\frac{6}{3}$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

a) $\left(8a^3b^6\right)^{1/3}$ $\frac{6}{1}\left(\frac{1}{3}\right)$

$=8^{1/3}a^{3\left(\frac{1}{3}\right)}b^2$

$=2ab^2$

b) $x^{\frac{4}{2}}y^1$

$=x^2y$

c) $\dfrac{4a^{-2}b^{2/3}}{2a^2b^{1/3}}$

$=2a^{-4}b^{1/3}$

$=\dfrac{2\sqrt[3]{b}}{a^4}$

d) $\left(\dfrac{100a^1}{25a^5b^{-1/2}}\right)^{1/2}$

$\left(\dfrac{4a^{-4}}{b^{-1/2}}\right)^{1/2}$

$=\dfrac{2a^{-2}}{b^{-1/4}}$

$=\dfrac{2b^{1/4}}{a^2}=\dfrac{2\sqrt[4]{b}}{a^2}$