Polynomials

By now, you should be familiar with variables and exponents. You may have dealt with expressions like $3x^4$ or $6x$. Polynomials are sums of these expressions. Each piece of the polynomial, each part that is being added, is called a "term". Polynomial terms have variables to whole-number exponents; there are no square roots of exponents, no fractional powers, and no variables in the denominator. Here are some examples:

$6x^{-2}$	NOT a polynomial term	This has a negative exponent.
$^1/_{x^2}$	NOT a polynomial term	This has the variable in the denominator.
$\sqrt{x}$	NOT a polynomial term	This has the variable inside a radical.
$4x^2$	a polynomial term	

Here is a typical polynomial:

terms

$$4x^2 + 3x - 7$$

- TERMS are separated by addition or subtraction signs.

Expanding and Simplifying:

• Collecting Like Terms

Probably the most common thing you will be doing with polynomials is "combining like terms". This is the process of adding together whatever terms you can, but not overdoing it by adding together terms that can't actually be combined.

Terms can be combined if they have the exact same variable part. Here is a rundown of what's what:

$4x$ and 3	NOT like terms	The second term has no variable
$4x$ and $3y$	NOT like terms	The second term now has a variable, but it doesn't match the variable of the first term
$4x$ and $3x^2$	NOT like terms	The second term now has the same variable, but the degree is different
$4x$ and $3x$	LIKE TERMS	Now the variables match and the degrees match

Examples:

1) $3x + 4x$

2) $2x^2 + 3x - 4 - x^2 + x + 9$

3) $10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$

4) $-4y - [3x + (3y - 2x + \{2y - 7\}) - 4x + 5]$

Ex.

Distrubitive Property

① Remove all Brackets ✓

② Collect Like Terms

$(w^2+3w-5)-1(2w^2-4w+1)$

$1w^2+3w-5-2w^2+4w-1$

$=-1w^2+7w-6$

2/ $(w^3-2w^2)-3(w^2+w)-5(1-3w^3)$

$= w^3-2w^2-3w^2-3w-5+15w^3$

$=16w^3-5w^2-3w-5$

3) $3w^2(4w^3-1w^1+7)$ $(3w^2)(4w^3)$

$=12w^5-3w^3+21w^2$

$(4w+5)^2$

$(4w+5)(4w+5)$

$16w^2\underbrace{+20w+20w}+25$

$16w^2+40w+25$

$(6w-1)^2$

$(6w-1)(6w-1)$

$=36w^2-6w-6w+1$

$=36w^2-12w+1$

Shortcut ... "3 Step Rule"

$(-21w)\times 2$

$(\underset{1^{st}}{3w}-\underset{2^{nd}}{7})^2$

Step 1: Square the first

Step 2: ($1^{st}\times 2^{nd}$) Doubled

Step 3: Square the 2^{nd}

$=9w^2-42w+49$

$(4w+5)^2$

$16w^2+40w+25$

$(3x-10)^2$

$3x(-10)(2)$

$-60x$

$= 9x^2-60x+100$

$(8x^4y^5-2x^3y)^2$

$=64x^8y^{10}-32x^7y^6+4x^6y^2$

• Distributive Property

The Distributive Property is easy to remember, if you recall that "multiplication *distributes* over addition and subtraction".

Formally, this property is displayed as...

$$a(b+c)=ab+ac$$

Examples:

1) $5x\left(2x^2-5\right)$

2) $2\left(7w^2-w\right)-3w(w+1)-\left(w^2-4w+2\right)$

3) $\left(4x-3y^3\right)\left(2x-y^2\right)$

$=8x^2-4xy^2-6xy^3+3y^5$

4) $(3w-2)^2$

$(3w-2)(3w-2)$

$=9w^2-6w-6w+4$

$=9w^2-12w+4$

5) $5(4w+3)^2$

$5(16w^2+24w+9)$

$=80w^2+120w+45$

6) $\left(2x^2-x+5\right)\left(3x^2+4x-1\right)$

$=6x^4+8x^3-2x^2-3x^3-4x^2+x+15x^2+20x-5$

$=6x^4+5x^3+9x^2+21x-5$

Practice Questions from Textbook:

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