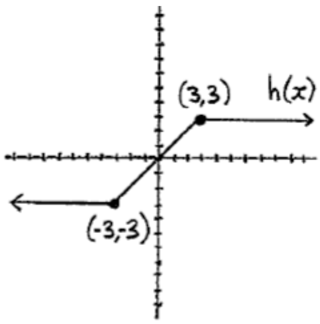
Warm-Up...



Given the graph of $h(x)$ above, match the following four functions with their graphs.

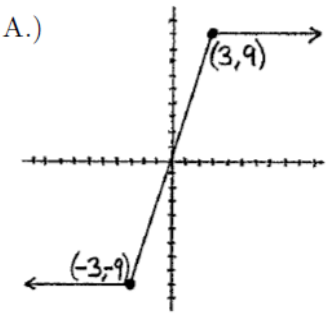
29.) $3h(x)$ A

30.) $\frac{1}{3}h(x)$ D

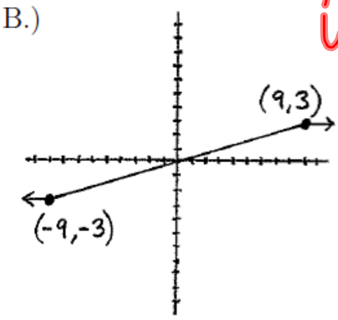
31.) $h(3x)$ C

B 32.) $h(\frac{x}{3})$ 3 times wider

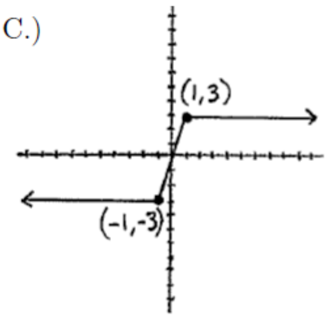
A.)



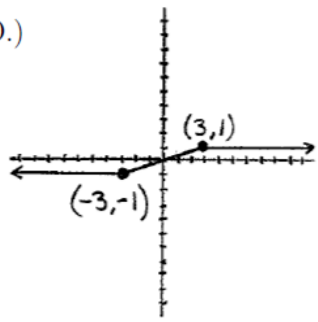
B.)



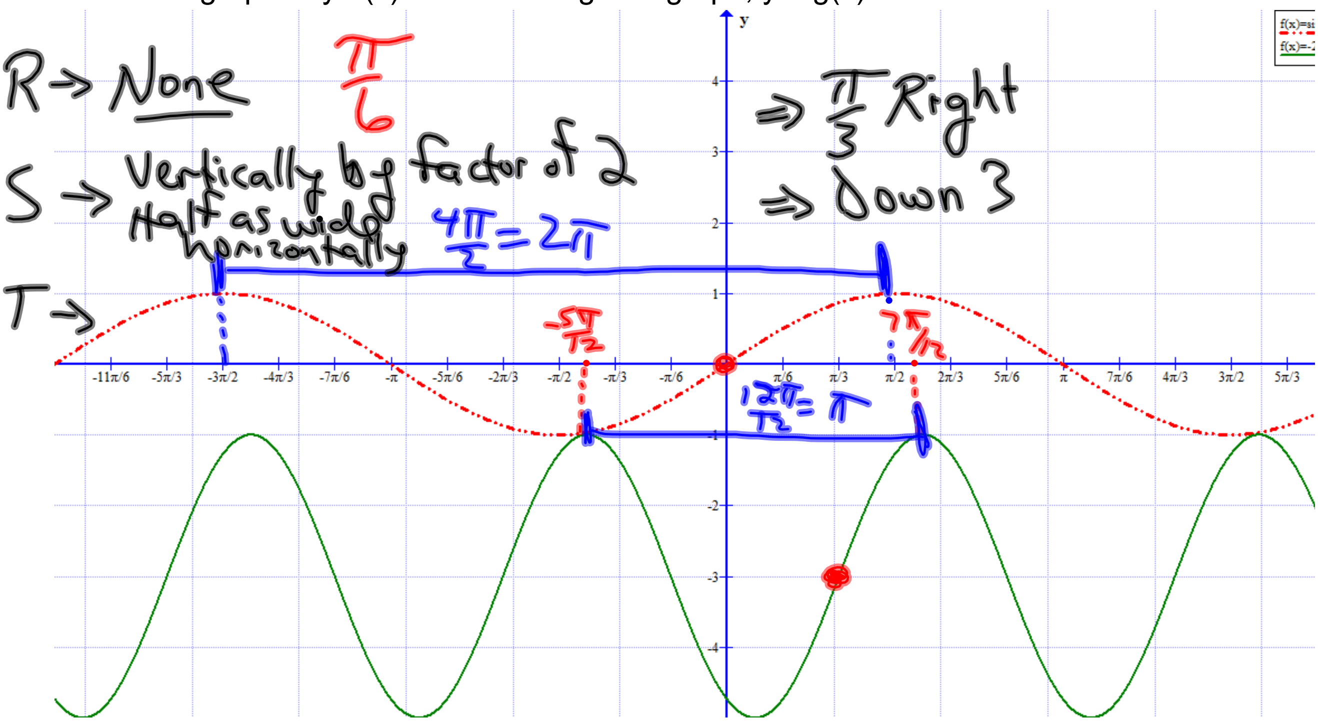
C.)



D.)



How did the graph of y=f(x) become the green graph, y = g(x) ??



$$g(x) = 2f\left(2\left(x - \frac{\pi}{3}\right)\right) - 3$$

Mapping:

$$(x, y) \rightarrow \left(\frac{1}{2}x + \frac{\pi}{3},\ 2y - 3\right)$$

b) $y-k=af(b(x-h))+k$

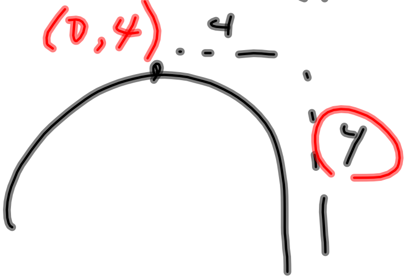
H. Stretch $\Rightarrow \frac{1}{3}$
V. Stretch $\Rightarrow \frac{3}{4}$
Reflects Both axes
6 units Right
2 units Up

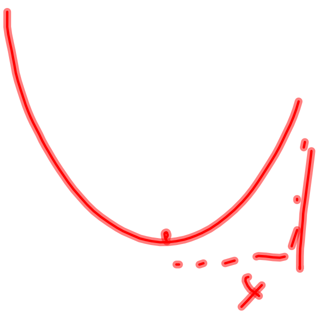
$$y=-\frac{3}{4}f(-3(x-6))+2$$

* R, S, T

10. a) Reflects in x-axis

$$y=-3f(x-8)+10$$





(0,-4)

(0,-12)

Inverse of a Relation

An inverse function is a second function which undoes the work of the first one.

1. Introduction

Suppose we have a function f that takes x to y, so that

$$f(x) = y.$$

An inverse function, which we call f^{-1}, is another function that takes y back to x. So

$$f^{-1}(y) = x.$$

For f^{-1} to be an inverse of f, this needs to work for every x that f acts upon.

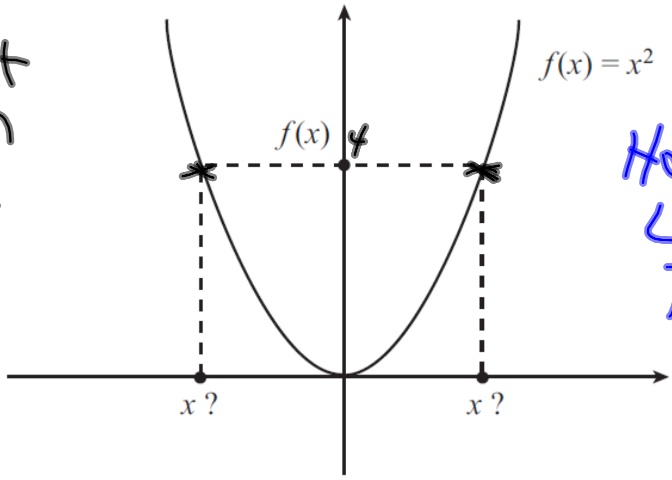
$8^{-1} = \frac{1}{8}$ $\quad x^{-1} = \frac{1}{x}$

Did You Know?

The –1 in $f^{-1}(x)$ does not represent an exponent; that is $f^{-1}(x) \neq \frac{1}{f(x)}$.

Not all functions have inverses. For example, let us see what happens if we try to find an inverse for $f(x) = x^2$.

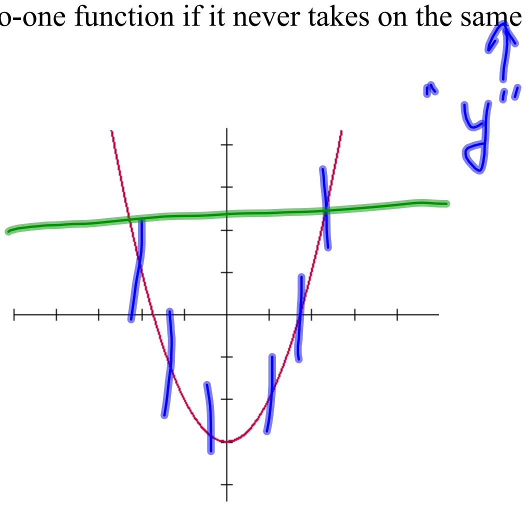
Does Not have an inverse



Horizontal Line Test . . .

A function is said to be a one-to-one function if it never takes on the same value twice.

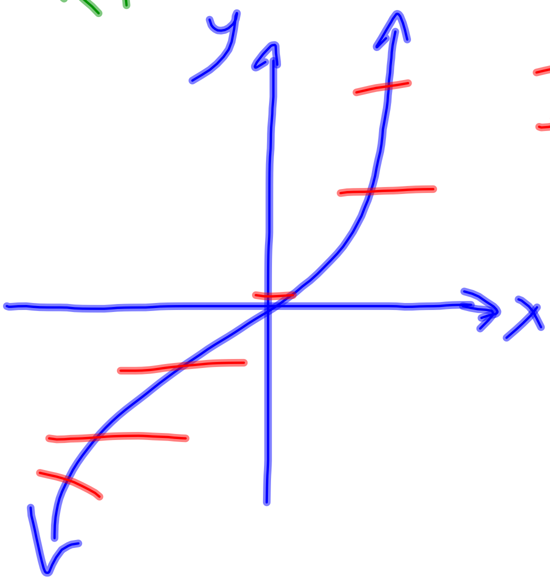
Look at this function...



Not a one-to-one function...

therefore

∴ Does not have an inverse



Is a one-one function ∴

Inverse Exists

If a function is a one-to-one function then it will posses what is called an inverse function.

If f is a one-to-one function with domain A and range B. Then its **inverse function,** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

for any y in B.

$$\boxed{\begin{array}{l} \text{domain of } f^{-1} = \text{range of } f \\ \text{range of } f^{-1} = \text{domain of } f \end{array}}$$

* In plain english....the x and y coordinates will just switch places

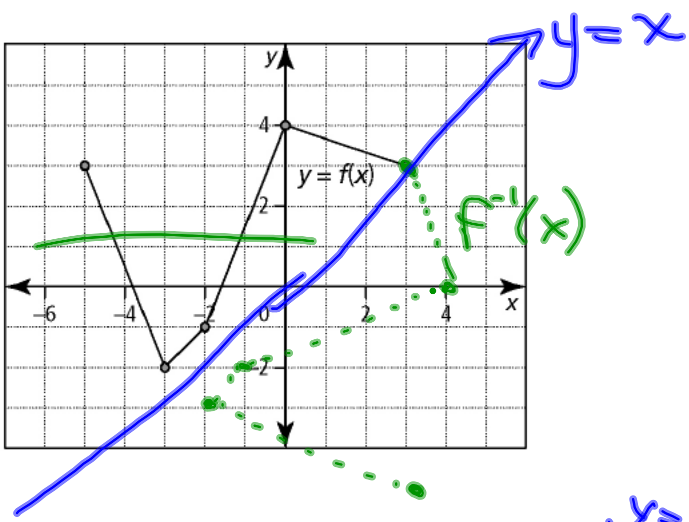
$f(x)$ $\qquad$ $f^{-1}(x)$

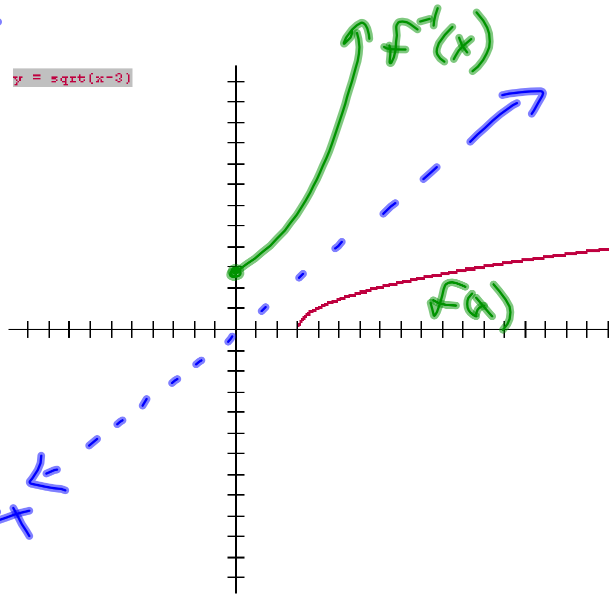
$(-7, 8) \Longrightarrow (8, -7)$

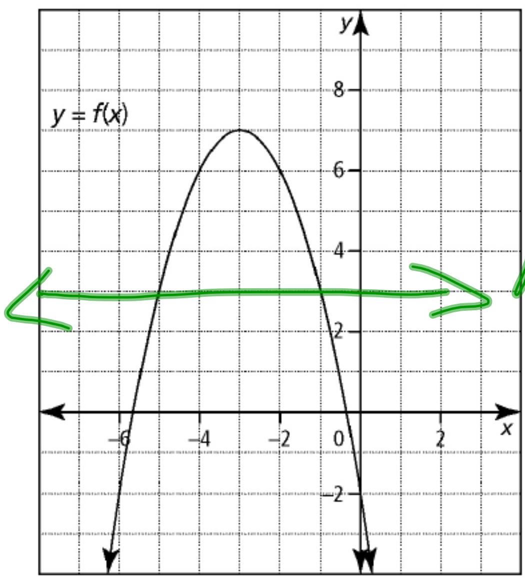
The inverse of a relation is found by interchanging the x-coordinates and y-coordinates of the ordered pairs of the relation. In other words, for every ordered pair (x, y) of a relation, there is an ordered pair (y, x) on the inverse of the relation. This means that the graphs of a relation and its inverse are reflections of each other in the line $y = x$.

$(x, y) \rightarrow (y, x)$

How does this play out graphically?







Not one-one

∴ No Inverse!!

What if given the function algebraically?

Determine algebraically the equation of the inverse of each function.

a) $f(x) = 3x - 6$ **b)** $f(x) = \frac{1}{2}x + 5$

$f^{-1}(x) =$

c) $f(x) = \frac{1}{3}(x + 12)$ ***d)** $f(x) = \frac{8x + 12}{4}$

a) $y = 3x - 6$

$x = 3y - 6$

$x + 6 = 3y$

$y = \frac{x+6}{3}$

$f^{-1}(x) = \frac{x+6}{3}$

x	y
0	2

x	y
2	0

d) $y = 2x + 3$

$x = 2y + 3$

$y = \frac{x-3}{2}$

$f^{-1}(x) = \frac{x-3}{2}$

Determine the inverse for each of the following functions:

1. $f(x) = 2x - 5$

2. $f(x) = \sqrt{x-3} + 4$

$x = \sqrt{y-3} + 4$

$x - 4 = \sqrt{y-3}$

$(x-4)^2 = y - 3$

$y = (x-4)^2 + 3$

$f^{-1}(x) = (x-4)^2 + 3$

Practice Problems...

Pages 51 - 55
#2, 3, 5, 6, 8, 9, 11, 15, 18, 20, 21