Ex. 1

Determine the coordinates of the point $(-3, 10)$ on $f(x)$, after the following transformations

$$g(x) = -3f\left[-\frac{2}{3}(x+4)\right] + 8$$

Reflect in x-axis

stretch vertically by a factor of 3

Reflect in y-axis

Horizontal stretch by factor of $\frac{3}{2}$

Moves 4 left

Moves up 8

$$(x, y) \rightarrow \left(-\frac{3}{2}x - 4,\ -3y + 8\right)$$

$$(-3, 10) \rightarrow \left(-\frac{3}{2}(-3) - 4,\ -3(10) + 8\right)$$

$$\rightarrow \left(\frac{1}{2},\ -22\right)$$

Pg. 39

$(-12, 18)$

#6 a) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$

$y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4$

$-6 \div -\frac{2}{3}$

$-6 \cdot -\frac{3}{2} = 9$

$3x + 15$

$3(x+5)$

$(x, y) \to \left(-\frac{3}{2}x - 9, -2y + 4\right)$

$(-12, 18) \to \left(-\frac{3}{2}(12) - 9, -2(18) + 4\right)$

$= (9, -32)$

#7) R S T

f) $3y-6=f(-2x+12)$

$\frac{3y}{3}=\frac{f(-2(x-6))}{3}+\frac{6}{3}$

$-\frac{2x}{-2}+\frac{12}{-2}$

$y=\frac{1}{3}f(-2(x-6))+2$

$-2(x-6)$

R:

Reflect in y-axis

S:

stretch vertically by factor of $\frac{1}{3}$

stretch horizontally by factor of $\frac{1}{2}$

T:

Right 6 & Up 2

Horizontal Shift?

$f\left(-\frac{3}{8}x-15\right)$

$-15\div-\frac{3}{8}$

$-\cancel{15}^{5}\cdot-\frac{8}{\cancel{3}}$

$f\left(-\frac{3}{8}(x+40)\right)$

"Left 40"

Pg. 55

#20 a) $f^{-1}(5)$; if $f(17)=5$

Inverse Functions $\Rightarrow$ "Switch x & y"

$f(17)=5$

$(17, 5) \Longrightarrow$ $f^{-1}(x)$ $(5, 17)$

c) $f^{-1}(a)=1$ & $f(x)=2x^2+5x+3$, $x \geq -1.25$

$(a, 1)$ is on inverse

$\therefore (1, a)$ is on $f(x)$

$a=2(1)^2+5(1)+3$

$\underline{a=10}$

21 c) $(10,8)$ is on $f(x)$

$$y = -f^{-1}(x) + 1$$

Inverse ... Mapping $\Rightarrow (x,y) \to (-x, -y+1)$

Switch x & y $\to (8,10) \to (-8, -10+1)$

$\to (-8, -9)$

Quiz

$\Rightarrow$ Sketching piecewise

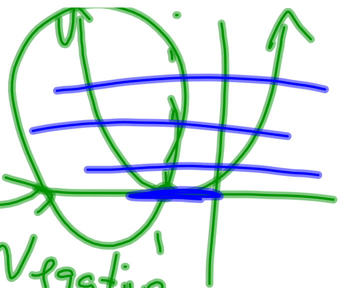
$\Rightarrow$ Transformations

$\Rightarrow$ Inverses

6. $y = \frac{1}{2}(x+1)^2$, $x \leq -1$

$\sqrt{25} = \sqrt{x^2}$

$\pm 5 = x$



Negative Side

$x = \frac{1}{2}(y+1)^2$

$\pm\sqrt{2x} = \sqrt{(y+1)^2}$

$-\sqrt{2x} = y+1$

$y = -\sqrt{2x} - 1$

$f^{-1}(x) = -\sqrt{2x} - 1$

Combination of Functions

- Two functions f and g can be combined to form new functions
  - $f+g$,
  - $f-g$,
  - fg, and
  - f/g

  just as we add, subtract, multiply, and divide real numbers.

- This is summarized in the following table:

Algebra of Functions Let f and g be functions with domains A and B. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows:

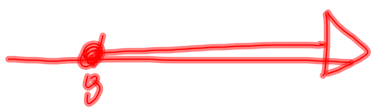
$(f+g)(x) = f(x) + g(x)$	domain $= A \cap B$ Intersects
$(f-g)(x) = f(x) - g(x)$	domain $= A \cap B$
$(fg)(x) = f(x)g(x)$	domain $= A \cap B$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	domain $= \{x \in A \cap B \mid g(x) \neq 0\}$

Set theory . . .

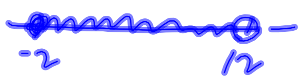
$A \cup B$ "Union of" Join together

$A \cap B$ Intersect Set A and B $\}$ overlap

$A: \; x \geqq 8, \; x \in R$

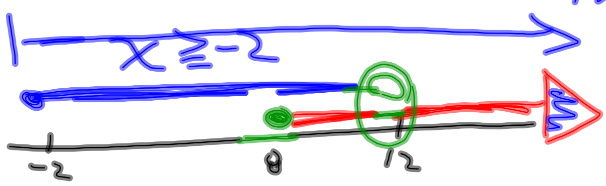


$B: \; -2 \leqq x < 12, \; x \in R$



$x \geqq -2$

$A \cup B \Rightarrow$



$A \cap B \Rightarrow 8 \leq x < 12, \; x \in R$

- ***Review of Intersection and Union of two sets:***

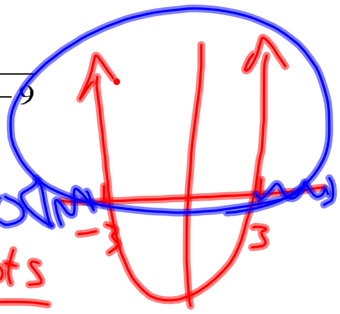
$f(x) = \sqrt{x+4}$ $\qquad\qquad$ $g(x) = \sqrt{x^2 - 9}$

Let A represent the domain of f and B the domain of g.

A: $x + 4 \geq 0$

$x \geq -4$

B: $x^2 - 9 \geq 0$



x-Intercepts

$x^2 - 9 = 0$

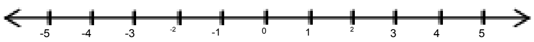
$(x-3)(x+3) = 0$

$x = \pm 3$

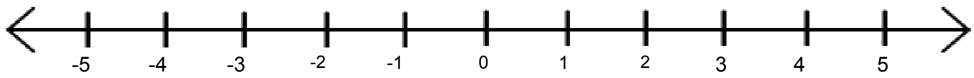
$x \leq -3$

or

$x \geq 3$



I. Intersection: $\quad A \cap B$



II. Union: $\quad A \cup B$