Example

- If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find the functions $f+g$, $f-g$, fg, and f/g.

**Also examine the domain of each of these new functions

$(f+g)(x)=\sqrt{x}+\sqrt{4-x^2}$

$(fg)(x)=\sqrt{x}\left(\sqrt{4-x^2}\right)$

$(f-g)(x)=\sqrt{x}-\sqrt{4-x^2}$

$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{\sqrt{4-x^2}}$

1. a) $f(-2) - f(1) + f(5)$

$f(-2) = -3$ $\quad$ $f(1) = -(1+1)^2 + 3$ $\quad$ $f(5) = 3(5) - 4$

$= -1$ $\quad$ $= 11$

$= -3 - (-1) + 11$

$= 9$

$y = -3$

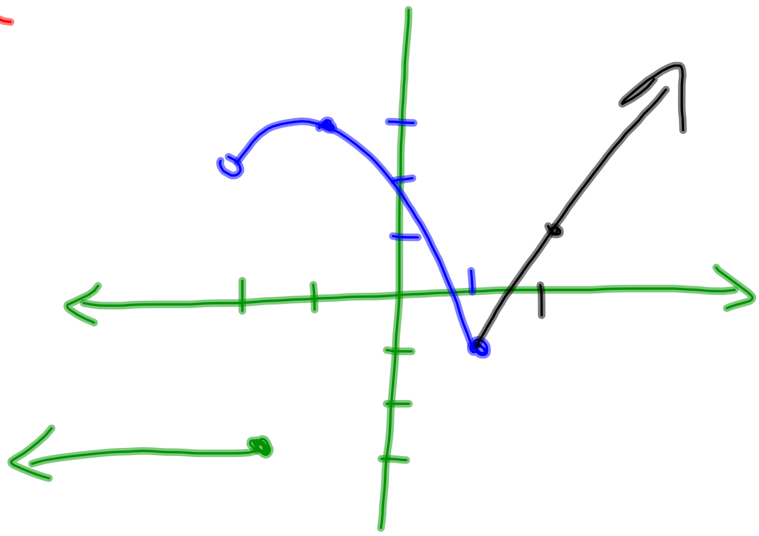
b) $y = -(x+1)^2 + 3$ $\quad$ $y = 3x - 4$

$V(-1, 3)$

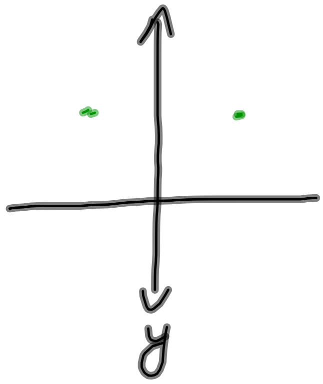
opens down

x	y
-2	2
1	-1

x	y
1	-1
2	2



2/ a) $g(x) = 3f\left[-\frac{5}{2}(x-5)\right] - 2$



OR

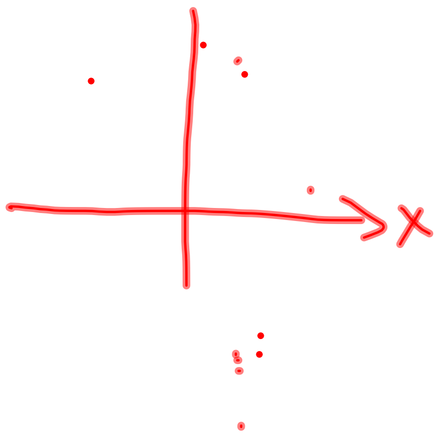
$g(x) = 3\sqrt{-\frac{5}{2}(x-5)} - 2$

b) $(x, y) \rightarrow \left(-\frac{2}{3}x + 5, 3y - 2\right)$

c) $(16, 4) \rightarrow \left(-\frac{2}{5}(16) + \frac{5}{1}, 3(4) - 2\right)$

$\rightarrow \left(\frac{-7}{5}, 10\right)$

3. $g(x) = 5f(-8(x-3)) - 7$



1) No
2) yes
3) Right 3
4) down 7
5) $\frac{1}{8}$
6) 5

4/ $y=2(x+3)^2-1 \quad , x \geq -3$

$x=2(y+3)^2-1$

$\frac{x+1}{2}=\frac{\cancel{2}(y+3)^2}{\cancel{2}}$

$\sqrt{\frac{x+1}{2}}=\sqrt{(y+3)^2}$

$\sqrt{\frac{x+1}{2}}=y+3$

$y=\sqrt{\frac{x+1}{2}}-3$

$f^{-1}(x)=\sqrt{\frac{x+1}{2}}-3$

$f^{-1}(17)=\sqrt{\frac{17+1}{2}}-3$

$=\sqrt{9}-3$

$=3-3=\underline{\underline{0}}$

5. Reflected in x-axis

V. Stretch = 6

left 2 up 5

$$g(x) = -6f(x+2)+5 = -f(6(x+2))+5$$

$$(x,y) \to (x-2, -6y+5)$$

$$(0,0) \to ((0)-2, -6(0)+5)$$

$$\to \underline{(-2,5)}$$

Compositions of Functions

When the input in a function is another function, the result is called a **composite function.** If

$f(x) = 3x + 2$ and $g(x) = 4x - 5$

then $f[g(x)]$ is a composite function. The statement $f[g(x)]$ is read " f of g of x" or "the composition of f with g." $f[g(x)]$ can also be written as

$$(f \circ g)(x) \quad \text{or} \quad f \circ g(x)$$

The symbol between f and g is a small open circle. When replacing one function with another, be very careful to get the order correct because compositions of functions are not necessarily commutative (as you'll see).

$f(2) = 8 \qquad g(-1) = -9$

$f[g(-1)]$

$f(-9) = 3(-9) + 2$

$= -25$

Example 1

If $f(x) = 3x + 2$ and $g(x) = 4x - 5$, find each of the following.

1. $f[g(4)] = \underline{\underline{35}}$
2. $g \circ f(4)$
3. $f[g(x)]$
4. $(g \circ f)(x)$

1) $g(4) = 11$

$f(11) = 35$

2) $f(4) = 14$

$g(14) = 56 - 5$

$= 51$

3) $f[g(x)]$

$= 3(g(x)) + 2$

$= 3(4x - 5) + 2$

$= 12x - 13$

4) $(g \circ f)x = g(f(x))$

$= 4(f(x)) - 5$

$= 4(3x + 2) - 5$

$= \underline{12x + 3}$

Example 2

If $f(x) = 3x^2 + 2x + 1$ and $g(x) = 4x - 5$, find each of the following:

1. $f[g(x)]$
2. $g[f(x)]$

$$f(g(x)) = 3(\overset{x^2}{4x-5})^2 + 2(\overset{x}{4x-5}) + 1$$

$$= 3(16x^2 - 40x + 25) + 8x - 10 + 1$$

$$= 48x^2 - 120x + 75 + 8x - 9$$

$$= \underline{48x^2 - 112x + 66}$$

$$g(f(x)) = 4(3x^2 + 2x + 1) - 5$$

$$= 12x^2 + 8x + 4 - 5$$

$$= \underline{12x^2 + 8x - 1}$$

Check Up

Given the three functions....

$f(x) = 1 - x$ $\quad$ $g(x) = \sqrt{x+1}$ $\quad$ $h(x) = x^2 + 5$

Evaluate each of the following:

1. $(f \circ g)(3)$ $= -1$
2. $(g \circ h)(0)$
3. $(g \circ g \circ f)(-7)$
4. $(h \circ g \circ h \circ f)(-1)$
5. $(f \circ h \circ g)(m)$

1/ $g(3) = \sqrt{4}$
$= 2$
$f(2) = 1 - 2$
$= -1$

2/ $h(0) = 0^2 + 5 = 5$
$g(5) = \sqrt{5+1} = \sqrt{6}$

3/ $f(-7) = 8$
$g(8) = \sqrt{9} = 3$
$g(3) = \sqrt{4} = 2$

4/ $f(-1) = 2$
$h(2) = 9$
$g(9) = \sqrt{10}$
$h(\sqrt{10}) = (\sqrt{10})^2 + 5$
$= \underline{15}$

5/ $g(m) = \sqrt{m+1}$
$h(\sqrt{m+1}) = (\sqrt{m+1})^2 + 5$
$= m + 6$
$f(m+6) = 1 - (m+6)$
$= -m - 5$

Unit Test: Monday

- ⇒ Sketch piecewise function
- ⇒ function Notation
  - ⇒ combinations: Domain (Intersection of Domains)
  - ⇒ compositions:
- ⇒ Catalog of essential functions

Notes

Functions: Transformations

→ Translations, Reflections, Stretches

$$y = a f[b(x-h)] + k$$

- a: -Vert. stretch, -Reflect in x-axis
- b: - Reflect in y-axis, - Horizontal stretch $\left(\frac{1}{b}\right)$
- h: Horizontal Translation
- k: Vertical Translation

Mapping:

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, \; ay + k\right)$$

⇒ Inverse Functions

- Switch "x" & "y" (Domain & Range)
- Sketch Inverses from a given graph (Reflects in line $y = x$)
- one-one function (Horizontal line)
- Switch to inverse algebraically

i.e. $f(x) = x + 7$

$x = y + 7$

$x - 7 = y$

$f^{-1}(x) = x - 7$

Chapter Review from textbook...

Pages 56-57
#2, 3, 6, 8, 9, 10, 11, 14, 15, 16

Practice Test
Pages 58-59
All questions