Determine a negative angle co-terminal with each of the following angles:

1) $476895° \Rightarrow$

Principal Angle (Must Be Positive)

$= -105°$

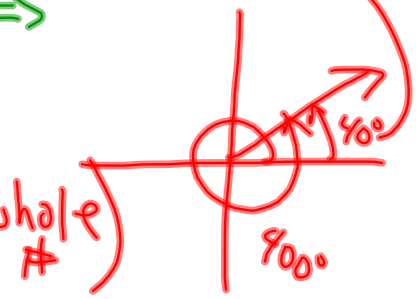
$\div 360° - (\text{whole } \#)$

$\times 360°$

$P.A. = 255°$

$-360°$

$-105°$



2) $\dfrac{35784\pi}{5}$

$= \dfrac{35784\pi}{5} - \dfrac{\pi}{5}$

$= 7157\pi - \dfrac{\pi}{5}$

direction (cw)

$\pi + \pi$

π

$\frac{5\pi}{5} + \frac{\pi}{5}$

$= \frac{6\pi}{5}$

$-\frac{6\pi}{5}$

odd π

even π

$\frac{\pi}{5}$

$\frac{5\pi}{5}$

0

$\frac{4\pi}{5}$

$\frac{6\pi}{5}$

Find the Principal Angle for each of the following:

1) 2583°

```
2583/360
                7.175
Ans-7
                 .175
Ans*360
                   63
```

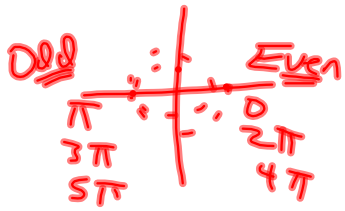
2) −37147°

```
                -103.1861111
Ans+103
                 -.1861111111
Ans*360
                        -67
Ans+360
                        293
```

3) $\frac{27\pi}{4}$

$\frac{28\pi}{4} - \frac{\pi}{4}$

$7\pi - \frac{\pi}{4}$

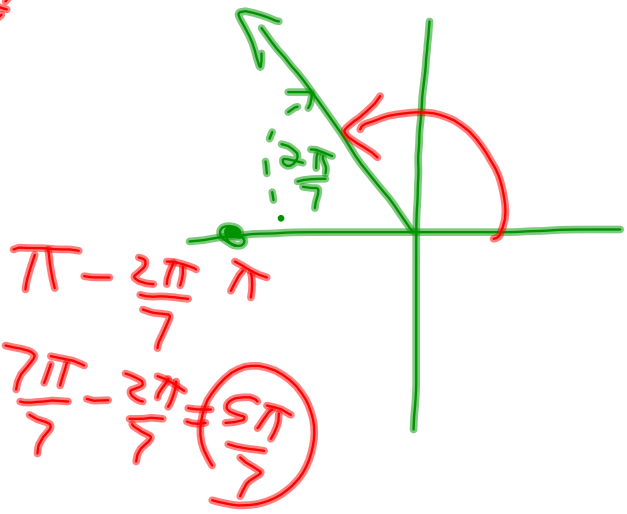


4) $\frac{-3159\pi}{7}$

$\frac{-3157\pi}{7} - \frac{2\pi}{7}$

$\frac{-451\pi}{} - \frac{2\pi}{7}$

odd



Check-Up...

Arrange the following angles in descending order:

$340°$ ① $\quad 4.28\,rad$ ③ $\quad \frac{9\pi}{5}$ ② $\quad (10\pi)°$ ④ $31.4°$

$4.28\,\text{Rad} \times \frac{180°}{\pi\,\text{Rad}} \approx 245°$

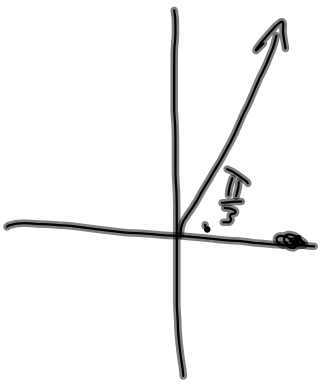
$\frac{9(180°)}{5} = 324°$

Determine a negative angle co-terminal with each of the following:

(i) $\frac{5881\pi}{3}$ $\quad -\frac{5\pi}{3}$

$\frac{5880\pi}{3} + \frac{\pi}{3}$

$1960\pi + \frac{\pi}{3}$ (even)



$\frac{\pi}{3} - \frac{2\pi}{1}$

$= \frac{\pi}{3} - \frac{6\pi}{3}$

$= -\frac{5\pi}{3}$

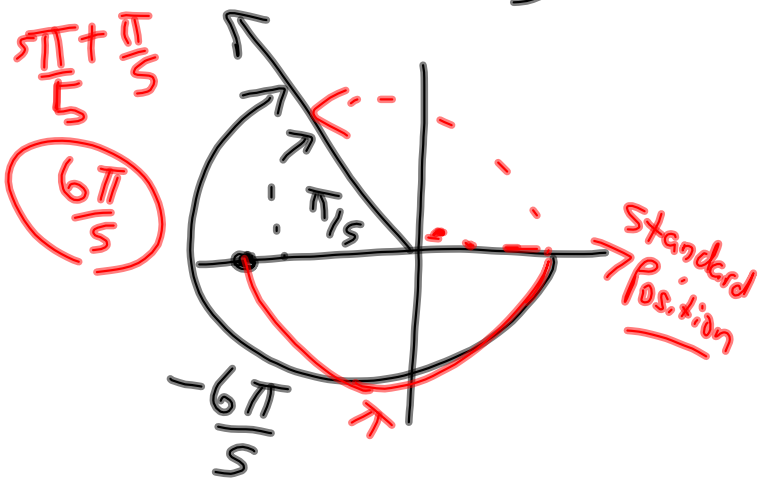
(ii) $\frac{29784\pi}{5}$ $\quad -\frac{6\pi}{5}$

$\frac{29785\pi}{5} - \frac{\pi}{5}$

$5957\pi - \frac{\pi}{5}$

$\frac{5\pi}{5} + \frac{\pi}{5}$

$\frac{6\pi}{5}$



Coterminal Angles in General Form

Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation from the terminal arm, you arrive back at the same terminal arm. Angles coterminal with any angle θ can be described using the expression

$\theta \pm (360°)n$ or $\theta \pm 2\pi n$,

where n is a natural number. This way of expressing an answer is called the **general form**.

general form

- an expression containing parameters that can be given specific values to generate any answer that satisfies the given information or situation
- represents all possible cases

Let's use the following two angles...

$\theta = 70°$ $\qquad$ $\theta = \frac{5\pi}{6}$

General Solution...

$\theta = 70° \pm 360°k, k \in N$
$\theta = 70° + 360°k, k \in I$
} General Solutions

Radians

$\theta = \frac{5\pi}{6} \pm 2\pi k, k \in N$
$= \frac{5\pi}{6} + 2\pi k, k \in I$
} General Solutions

What if we are given a restricted domain?

$\theta = 70°$, $-720° \le \theta \le 1080°$

$\theta = 420°, 780°,$
$-290°, -650°$

$\theta = \frac{5\pi}{6}$, $-2\pi \le \theta \le 8\pi$

$1 + 12\pi$
6

48π
6

$\theta = \frac{17\pi}{6}, \frac{29\pi}{6}, \frac{41\pi}{6}$

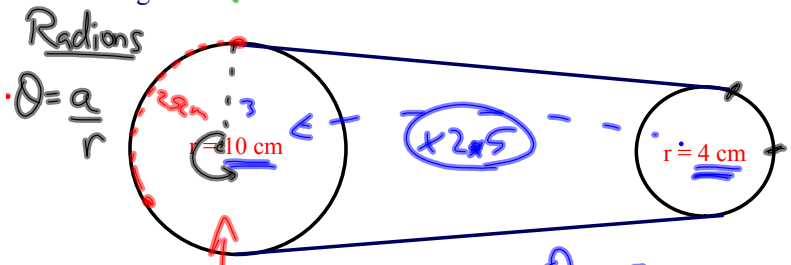
$= -\frac{7\pi}{6}$

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger wheel rotate?



Radians

$\theta = \frac{a}{r}$

$\theta = \frac{2\pi}{3}$

$\theta = \frac{a}{r}$

$(10)\frac{2\pi}{3} = \frac{a}{10\,cm}(10)$

$a = \frac{20\pi}{3}\,cm$

$\theta = \frac{a}{r}$

$\theta = \left(\frac{20\pi}{3}\right)cm \,/\, 4\,cm$

$\theta = \left(\frac{20\pi}{3}\cdot\frac{1}{4}\right)Rad$

$\theta = \frac{5\pi}{3}\,Rad$

Ratios: (2.5)

Big $\Rightarrow$ Small ($\times 2.5$)

$\theta = \frac{2\pi}{3} \times 2.5 = \frac{5\pi}{3}$

b) Large Wheel $\xrightarrow{\times 2.5}$ Smaller

3 Revolutions

$6\pi\,Rad \times 2.5 = 15\pi\,Rad$

c) $-\frac{15\pi}{4}$ small $\Rightarrow$ Large ($\div 2.5$)

$-\frac{15\pi}{4} \times \frac{1}{2.5} = -\frac{6\pi}{4} = -\frac{2\pi}{3}$

Practice Problems...

Pages 175 - 178
#3, 4, 5, 6, 7, 9, 11, 12, 13