Sequences and Series

Sequence:

A pattern of numbers in a definite order that follow a certain rule.

Examples of sequences:

1) 1, 2, 3, 4, 5, 6, 7, ... *add 1 to the preceding term*
2) 2, 4, 7, 11, 16, 23, 31. *add 2 to the preceding term, add 3 to the next term, etc*
3) 1, 1, 2, 3, 5, 8, 13, 21, 34,... *add the two preceding terms together*

Series:

The sum of the terms in a sequence.

Using the above sequences, we have the following series:

1) 1 + 2 + 3 + 4 + 5 + 6 + 7 +...
2) 2 + 4 + 7 + 11 + 16 + 23 + 31.
3) 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 +...

Finite Sequence or Series:

Comes to a definite end

2 + 4 + 7 + 11 + 16 + 23 + 31

Infinite Sequence or Series:

Continue indefinitely

1, 1, 2, 3, 5, 8, 13, 21, 34,...

Notation of Sequences and Series:

-5, 0, 5, 10, 15, 20, ... → $t_4 = 10$.

- Each element of a sequence or series is referred to as a term.

$t_1, t_2, t_3, t_4, t_5, t_6, \ldots, t_{n-1}, t_n$ ← Referred to as the n^{th} term or the General Term.

General Term:

An equation or formula used to determine the values of the terms in a sequence.

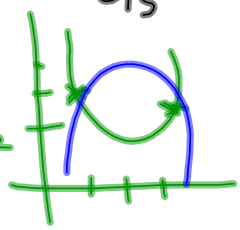
Example: $t_n = 5n - n^2$ What is the 15th term in this sequence?

$t_1 = 4$, $t_2 = 6$, $t_3 = 6$, $t_4 = 4$

4, 6, 6, 4, ...

$t_{15} = 5(15) - (15)^2 = -150$

$t_1 = 5(1) - (1)^2 = 4$



t_{15}

$f(x) = 2x - 3$

$f(3) = 2(3) - 3 = 3$

Recursive Sequence:

A sequence that uses the previous term to come up with each successive term.

- Must be given the first term to develop a recursive sequence.

Example: $t_1 = 4$

$t_n = -2n + 4(t_{n-1} + 1)$ What is the 5th term in this sequence?

$t_2 = -2(2) + 4(4+1) = 16$

$t_3 = -2(3) + 4(16+1) = 62$

$t_4 = -2(4) + 4(62+1) = 244$

$t_5 = -2(5) + 4(244+1) = 970$

Examples:

1. Determine the fifth and tenth terms of the sequence defined by the following general term: $t_n = 20n - 5(1 - n^2)$

$t_5 = 20(5) - 5(1 - 5^2) = 220$ 4th

$t_{10} = 20(10) - 5(1 - 10^2) = 695$

2. Determine the ~~eighth~~ term of the recursive sequence with an initial term of -5 and defined by the general term: $t_n = 5t_{n-1} + 2n$