Warm Up

1. During a recent home renovation Abby finds an old financial statement that belonged to her grandfather. The document has faded and some of the dates are difficult to read. She can see that her grandfather deposited $10000 on June 30, 1944. The balance of the investment at the time of this statement is $107651.63, however, the date has been smudged out. If this money was invested at 8%/a and compounded quarterly, what must have been the date on this statement?

$\$10\,000 \xrightarrow{\times 1.02} \cdots \; \$107651.63$

$t_{??}$

$t_n = ar^{n-1}$

$$\frac{107651.63}{10000} = \frac{\cancel{10000}(1.02)^{n-1}}{\cancel{10000}}$$

$$10.765163 = 1.02^{n-1}$$

$$\log 10.765163 = \log 1.02^{n-1}$$

$$\log 10.765163 = (n-1)\log 1.02$$

$$\frac{\log 10.765163}{\log 1.02} + 1 = n$$

$$120.999\ldots = n$$

$$n = 121$$

Interest was added on 120 times... (120 3 month intervals)

$120 \div 4 = 30$ years

Date: June 30, 1974

2. "Lenny the loan shark" gives out a $500 loan to a client. Lenny lends the money at 32%/a simple interest, and demands that he is repaid in 30 days...OR ELSE!!! How much must be paid to "Lenny the loan shark" 30 days later?

$$I = Prt$$

$$= 500(0.32)\left(\frac{30}{365}\right)$$

$$= \$13.15$$

Owe Lenny \$513.15

Questions from homework?

12 intervals

3\. $1, 2, 4, 8, \ldots t_{??}$

12 intervals $\longrightarrow t_{13}$

$t_{13} = (1)(2)^{12}$

6\. $400, 480 \ldots \longrightarrow t_7 = ?$

After 1 year

$\times 1.20$

$t_7 = 400(1.2)^6$

Arithmetic

#5/ 18 500, 20 000

After 1 year

$t_{13} = 18\,500 + (12)(1500)$

$= \$36\,500$

Geometric

#8/ $t_3 = 220\,000$ $t_6 = 292\,820$

$ar^2 = 220\,000$ $ar^5 = 292\,820$ $t_{10} = ?$

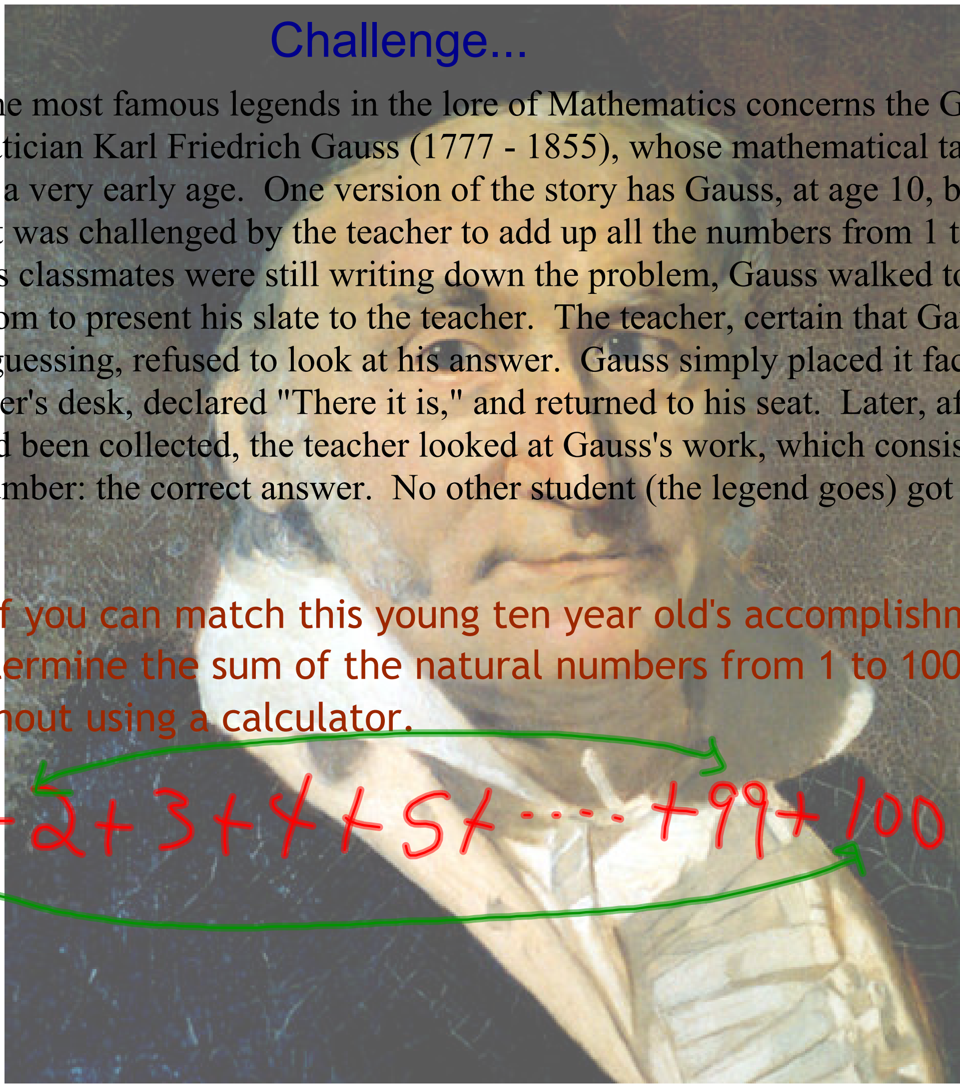
$\frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{292\,820}{220\,000}$

$\sqrt[3]{r^3} =$

$r = 1.1 \Longrightarrow a(1.1)^2 = 220\,000$

$t_{10} = 181\,818(1.1)^9$ $a = 181\,818$

$= 428\,718$



Challenge...

One of the most famous legends in the lore of Mathematics concerns the German mathematician Karl Friedrich Gauss (1777 - 1855), whose mathematical talent was apparent a very early age. One version of the story has Gauss, at age 10, being in a class that was challenged by the teacher to add up all the numbers from 1 to 100. While his classmates were still writing down the problem, Gauss walked to the front of the room to present his slate to the teacher. The teacher, certain that Gauss could only be guessing, refused to look at his answer. Gauss simply placed it face down on the teacher's desk, declared "There it is," and returned to his seat. Later, after all the slates had been collected, the teacher looked at Gauss's work, which consisted of a single number: the correct answer. No other student (the legend goes) got it right.

See if you can match this young ten year old's accomplishment...

- Determine the sum of the natural numbers from 1 to 100 without using a calculator.

$1+2+3+4+5+\cdots+99+100$

Series

Summation Notation:

The capital Greek letter sigma $\sum$ is used to provide a shorthand notation for a summation.

In summation notation, the sum of the terms of the sequence $\{a_1, a_2, a_3, \ldots, a_n\}$

is denoted $\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$

which is read "the sum of a_k from $k = 1$ to n"

Sigma notation is actually more widely used than the above definition suggests.

See if you can evaluate each of the following summations:

$$\sum_{k=1}^{5} 3k = \overset{\substack{k=1 \\ 3(1)}}{3} + \overset{\substack{k=2 \\ 3(2)}}{6} + 9 + 12 + 15$$

$$\sum_{k=4}^{9} (k-2) = \overset{k=4}{2} + \overset{k=5}{3} + \cdots 4 + 5 + 6 + 7$$

$$\sum_{k=1}^{5} \left(k^2 - 3k\right) = -2 + -2 + 0 + 4 + 10$$

Arithmetic Series:

- The summation of the terms of an arithmetic sequence

Formula $\dashrightarrow$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Let's derive this formula that can be used to determine the sum of "n" terms in any arithmetic series...

n-terms

$$S = a + (a+d) + (a+2d) + (a+3d) + \ldots + [a+(n-2)d] + [a+(n-1)d]$$

$$S = 2a+(n-1)d + 2a+(n-1)d + \text{- - - -} + 2a+(n-1)d$$

$$S = \frac{n}{2}\left[2a+(n-1)d\right]$$

Example:

Evaluate the following summation:

$$-2+7+16+25+34+...+1420=$$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{159} = \frac{159}{2}[2(-2)+(158)(9)]$

$= 112731$

$t_n = a+(n-1)d$

$1420 = -2+(n-1)9$

$1420 = -2+9n-9$

$1431 = 9n$

$\underline{n = 159}$

Example:
A corner section of a stadium has 8 seats along the front row. Each successive row has two more seats than the row preceding it. If the top row has 24 seats, how many seats are in the entire section?

Pg. 27

#1, 5, 6, 7,

Attachments

applications of sequences.doc