Factor fully:

Trial & Error

$6w^3+13w^2+2w-5$

FACTOR THEOREM

$w=\pm1,\pm5$

$w=-1$

$-6+13-2-5=0$

$\therefore (w+1)$ is a factor

$6w^3+13w^2+2w-5$

$w+1$??

$$\begin{array}{r|rrrr} 1 & 6 & 13 & 2 & -5 \\ & & 6 & 7 & -5 \\ \hline & 6 & 7 & -5 & 0 \end{array}$$

$(w+1)(6w^2+7w-5)$

$(w+1)\left(\frac{6w+10}{2}\right)\left(\frac{6w-3}{3}\right)$

$\boxed{(w+1)(3w+5)(2w-1)}$

$x^4-3x^3-7x^2+15x+18$

$x=-1$

$1+3-7-15+18=0$

$\therefore (x+1)$ is a factor

$$
\begin{array}{r|rrrrr}
1 & 1 & -3 & -7 & 15 & 18 \\
 & & 1 & -4 & -3 & 18 \\
\hline
 & 1 & -4 & -3 & 18 & 0
\end{array}
$$

$(x+1)(x^3-4x^2-3x+18)$

$\hookrightarrow$ $x=-2$

$-8-16+6+18=0$

$\therefore (x+2)$ is a factor

$$
\begin{array}{r|rrrr}
2 & 1 & -4 & -3 & 18 \\
 & & 2 & -12 & 18 \\
\hline
 & 1 & -6 & 9 & \underline{\underline{0}}
\end{array}
$$

$(x+1)(x+2)(x^2-6x+9)$

$(x+1)(x+2)(x-3)^2$

Factor

$x^4-3x^3-7x^2+15x+18$

$x=3$

$81-81-63+45+18=0$

$(x-3)$ is factor

$$\begin{array}{r|rrrrr} -3 & 1 & -3 & -7 & 15 & 18 \\ & & -3 & 0 & 21 & 18 \\ \hline & 1 & 0 & -7 & -6 & \underline{\underline{0}} \end{array}$$

$(x-3)(x^3-7x-6)$

$\hookrightarrow x={}^-1$

$-1+7-6=0$

$\therefore (x+1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & -6 \\ & & 1 & -1 & -6 \\ \hline & 1 & {}^-1 & -6 & 0 \end{array}$$

$(x-3)(x+1)(x^2-x-6)$

$$\boxed{(x-3)(x+1)(x-3)(x+2)}$$

Solving Polynomial Equations

The Factor Theorem can be used to identify points where the graph of a function crosses the x-axis.

Solve $16x^3 + 48x^2 - x - 3 = 0$

$x = -3$

$\therefore (x+3)$ is factor

$$\begin{array}{r|rrrr} 3 & 16 & 48 & -1 & -3 \\ & & 48 & 0 & -3 \\ \hline & 16 & 0 & -1 & 0 \end{array}$$

$(x+3)(16x^2-1)=0$

$(x+3)(4x-1)(4x+1)=0$

$x = -3, \pm\frac{1}{4}$

Ex. Identify the points where the graph of the function $f(x) = x^3 + 6x^2 - 2x - 5$ crosses the x-axis.

$x = 1$ $\quad 1 + 6 - 2 - 5 = 0$ ✓

$\therefore (x-1)$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & 6 & -2 & -5 \\ & & -1 & -7 & -5 \\ \hline & 1 & 7 & 5 & 0 \end{array}$$

$$(x-1)(x^2+7x+5) = 0$$

$\longrightarrow x^2 + 7x + 5 = 0$

$$x = \frac{-7 \pm \sqrt{49 - 4(1)(5)}}{2}$$

$$x = \frac{-7 \pm \sqrt{29}}{2}$$

$$x = \frac{-7}{2} \pm \frac{\sqrt{29}}{2}$$

x-Intercepts:

$x = 1$

$x = \frac{-7}{2} + \frac{\sqrt{29}}{2}$

$x = \frac{-7}{2} - \frac{\sqrt{29}}{2}$

Sum and Difference of Cubes

$$\sqrt[3]{x^{36}} \qquad (x^{36})^{1/3} = x^{12}$$

$$x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$$

Write binomial factor (keep sign!)

First term of binomial squared

First x Second (Change Sign!)

Second term of binomial squared

$$Ex \quad \left(8x^3 - 27\right)$$

$$(2x-3)\underbrace{(4x^2+6x+9)}_{\text{Always has non-Real Roots}}$$

$$Ex.\ 432x^6 + 2y^3$$

$$2(216x^6+y^3)$$

$$2(6x^2+y)(36x^4-6x^2y+y^2)$$

Interesting... Factor fully!!

$$(w^{12}-1)$$

$$(w^6-1)(w^6+1)$$

$$(w^3-1)(w^3+1)(w^6+1)$$

$$\boxed{(w-1)(w^2+w+1)(w+1)(w^2-w+1)(w^2+1)(w^4-w^2+1)}$$

OR

Start with diff. of cubes..

$$(w^4-1)(w^8+w^4+1)$$

$$(w^2-1)(w^2+1)(w^8+w^4+1)$$

$$\boxed{(w-1)(w+1)(w^2+1)(w^8+w^4+1)}$$

Practice:

Pg. 134/135

5, 6, 7, 13, 15, 16

Pg. 124 & 125

#14/ mx^3-3x^2+nx+2

1) $\div(x+3)$ Rem $= -1$
2) $\div(x-2)$ Rem $= -4$

$\left.\right\}$ $m=?$ $n=?$

sub. $x=-3$

$m(-3)^3-3(-3)^2+n(-3)+2=-1$

$-27m-27-3n+2=-1$

① $-27m-3n=24$

sub. $x=2$

$m(2)^3-3(2)^2+n(2)+2=-4$

$8m-12+2n+2=-4$

② $8m+2n=6$

$4m+n=3$

$n=3-4m \Longrightarrow -27m-3(3-4m)=24$

$-15m-9=24$

$-15m=33$

$m=\frac{-33}{15}=-\frac{11}{5}$

$n=\frac{3}{1}-4\left(-\frac{11}{5}\right)$

$n=\frac{59}{5}$