KANSAS STATE UNIVERSITY

Let's borrow a few examples from final exams given to the Wildcats over the past years...

Evaluate $\int \frac{(x+2)(x+4)}{(x+1)(x+3)(x+5)}\,dx$.

$$\int \frac{x^3+2x-1}{x^2-1}\,dx$$

$d(b^u) = b^u \ln b\,(du)$

Evaluate $\int \frac{dx}{x^2+6x+13}$

Evaluate $\int_0^1 2^{-x}\,dx$.

$$\int \frac{\sqrt{x^2-1}}{x}\,dx$$

Evaluate $\int \frac{\cos^5(x)}{\sin^2(x)}dx$.

Evaluate $\int \frac{(x+2)(x+4)}{(x+1)(x+3)(x+5)}\,dx$.

$$\frac{A}{x+1}+\frac{B}{x+3}+\frac{C}{x+5}=\frac{x^2+6x+8}{\cdots}$$

$$A(x+3)(x+5)+B(x+1)(x+5)+C(x+1)(x+3)=x^2+6x+8$$

$$A(x^2+8x+15)+B(x^2+6x+5)+C(x^2+4x+3)=x^2+6x+8$$

$$Ax^2+8Ax+15A+Bx^2+6Bx+5B+Cx^2+4Cx+3C=x^2+6x+8$$

$A+B+C=1$ $\quad 8A+6B+4C=6$ $\quad 15A+5B+3C=8$

$A=1-B-C$

$15(1-B-C)+5B+3C=8$

$15-15B-15C+5B+3C=8$

$-10B-12C=-7$

$10B+12C=7$

$4(1-B-C)+3B+2C=3$

$4-4B-4C+3B+2C=3$

$-B-2C=-1$

$B+2C=1$

$10B+12C=7$

$10B+20C=10$

$-8C=-3$

$C=\frac{3}{8}$

$B+\frac{2}{1}\left(\frac{3}{8}\right)=1$

$B+\frac{6}{8}=1$

$B=\frac{8}{8}-\frac{6}{8}=\frac{1}{4}$

$A=1-\frac{1}{4}-\frac{3}{8}$

$A=\frac{8}{8}-\frac{2}{8}-\frac{3}{8}$

$A=\frac{3}{8}$

$$\frac{3}{8}\int\frac{dx}{x+1}+\frac{1}{4}\int\frac{dx}{x+3}+\frac{3}{8}\int\frac{dx}{x+5}$$

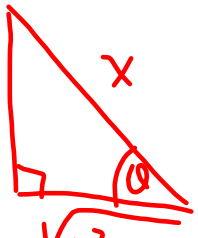
$$\frac{3}{8}\ln|x+1|+\frac{1}{4}\ln|x+3|+\frac{3}{8}\ln|x+5|+C$$

$$\int \frac{x^3+2x-1}{x^2-1}\,dx$$

$$x^2-1 \overline{)\, x^3+2x-1} \quad \begin{array}{r} x \\ \hline \\ x^3-x \\ \hline 3x-1 \end{array}$$

$$\int x\,dx + \int \frac{3x-1}{x^2-1}\,dx$$

$$\int x\,dx + \frac{3}{2}\int \frac{2x\,dx}{x^2-1} - \int \frac{dx}{x^2-1}$$




$$+\int \frac{\csc\theta\cot\theta\,d\theta}{\cot^2\theta} \qquad \cot\theta = \sqrt{x^2-1}$$

$$+\int \frac{\csc\theta}{\cot\theta}\,d\theta \qquad \cot^2\theta = x^2-1$$

$$\csc\theta = x$$

$$-\csc\theta\cot\theta\,d\theta = dx$$

$$+\int \frac{\left(\frac{1}{\sin\theta}\right)}{\left(\frac{\cos\theta}{\sin\theta}\right)}\,d\theta \qquad \left(\frac{1}{\sin\theta}\right)\times\frac{\sin\theta}{\cos\theta}$$

$$+\int \sec\theta\,d\theta$$

$$+\int \sec\theta\,d\theta\left(\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right)$$

$$+\ln|\sec\theta+\tan\theta|$$

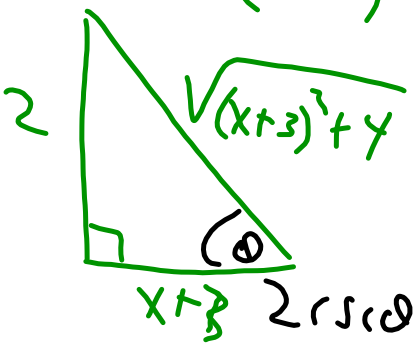
$$= \frac{x^2}{2} + \frac{3}{2}\ln|x^2-1| + \ln\left|\frac{x}{\sqrt{x^2-1}} + \frac{1}{\sqrt{x^2-1}}\right| + C$$

Evaluate $\int \frac{dx}{x^2+6x+13}$

$\int \frac{dx}{(x+3)^2+4}$

$(x^2+6x+\underline{9})+13-9$

$(x+3)^2+4$




$2\csc\theta = \sqrt{(x+3)^2+4}$

$4\csc^2\theta = (x+3)^2+4$

$2\cot\theta = x+3$

$-2\csc^2\theta\, d\theta = 1\, dx$

$-\frac{2}{4}\int \frac{\csc^2\theta\, d\theta}{\csc^2\theta}$

$-\frac{1}{2}\int d\theta$

$-\frac{1}{2}\theta + C$

$-\frac{1}{2}\cot^{-1}\left(\frac{x+3}{2}\right)+C$

Evaluate $\int_0^1 2^{-x}\,dx$.

$$\frac{-1}{\ln 2}\int_0^1 2^{-x}(\ln 2)(-1)\,dx$$

$$\frac{-1}{\ln 2}\left(2^{-x}\right)\Big|_0^1$$

$$\frac{-1}{\ln 2}\left(2^{-1}-2^{0}\right)$$

$$\frac{-1}{\ln 2}\left(\frac{1}{2}-1\right)$$

$$= \frac{1}{2\ln 2} = \frac{1}{\ln 4} = (\ln 4)^{-1}$$

$$d(b^u) = b^u \ln b\, du$$

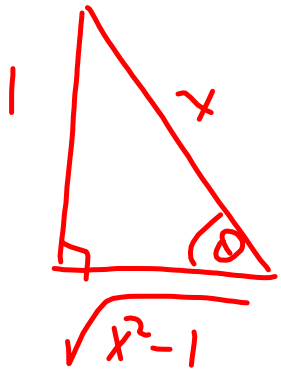
$u = -x$

$du = -1\,dx$

$-\int 2^u\,du$

$x \ln b = \ln b^x$

$$\int \frac{\sqrt{x^2-1}}{x}\,dx$$




$\cot\theta = \sqrt{x^2-1}$

$\csc\theta = x$

$-\csc\theta\cot\theta\,d\theta = dx$

$\int \frac{\cot\theta(-\csc\theta\cot\theta)\,d\theta}{\csc\theta}$

$-\int \cot^2\theta\,d\theta$

$-\int(\csc^2\theta - 1)\,d\theta$

$+\cot\theta + \theta + C$

$+\sqrt{x^2-1} + \csc^{-1}x + C$

Evaluate $\int \frac{\cos^5(x)}{\sin^2(x)} dx$.

$$\frac{\cos x \cos^4 x}{\sin^2 x} \frac{(1-\sin^2 x)^2}{\sin^2 x} \cos x$$

***John Abbott College: May and December 2011 Final Exam Questions***

**Set A**

2. Evaluate the following integrals.

(3) (a) $\int \frac{x^2}{\sqrt{x-4}}dx$

(4) (b) $\int \frac{x \arcsin(x^2)}{\sqrt{1-x^4}}dx$

(4) (c) $\int_0^{\pi/4} \sqrt{\tan x}\sec^4 x dx$

(5) (d) $\int (\cos^2\theta + \sin^3\theta)d\theta$

(4) (e) $\int \frac{\sqrt{9x^2-4}}{x}dx$

(4) (f) $\int \frac{6x^2-5x-1}{(x-2)(x^2+9)}dx$

(4) (g) $\int 16x(\arctan(4x))dx$

**Set B**

(5) (a) $\int_1^5 \frac{x+2}{\sqrt{2x-1}}dx$

(5) (b) $\int \frac{1}{x^3\sqrt{x^2-4}}dx$

(5) (c) $\int \frac{\tan^{-1}x}{x^2}dx$

(5) (d) $\int \frac{\sec^4\sqrt{x}\,\tan^2\sqrt{x}}{\sqrt{x}}dx$

(5) (e) $\int_0^{\frac{1}{2}} \frac{x+\arccos x}{\sqrt{1-x^2}}dx$

(5) (f) $\int \frac{e^x}{\sqrt{3-2e^x-e^{2x}}}dx$

(5) (g) $\int \frac{3x^2-2}{x^2-2x-8}dx$

Set A

2. (a) $\frac{2}{5}(x-4)^{5/2}+\frac{16}{3}(x-4)^{3/2}+32(x-4)^{1/2}+C$

(b) $\frac{(\arcsin(x^2))^2}{4}+C$

(c) $\frac{20}{21}$

(d) $\frac{1}{2}\left(\theta+\frac{\sin(2\theta)}{2}\right)-\cos\theta+\frac{\cos^3\theta}{3}+C$

(e) $\sqrt{9x^2-4}-2\text{arcsec}\left(\frac{3x}{2}\right)+C$

(f) $\ln|x-2|+\frac{5}{2}\ln(x^2+9)+\frac{5}{3}\arctan\left(\frac{x}{3}\right)+C$

(g) $\frac{(16x^2+1)\arctan(4x)}{2}-2x+C$

Set B

**Answers**

1.(a) $\frac{2}{9}$ (b) 4 2.(a) $\frac{28}{3}$ (b) $\frac{1}{16}\left(\sec^{-1}\left(\frac{x}{2}\right)+\frac{2\sqrt{x^2-4}}{x^2}\right)+C$

(c) $-\frac{\tan^{-1}x}{x}+\ln x-\frac{1}{2}\ln(x^2+1)+C$ (d) $2\left(\frac{\tan^5\sqrt{x}}{5}+\frac{\tan^3\sqrt{x}}{3}\right)+C$ (e) $1-\frac{\sqrt{3}}{2}+\frac{5\pi^2}{72}$

(f) $\arcsin\left(\frac{e^x+1}{2}\right)+C$ (g) $3x+\frac{23}{3}\ln|x-4|-\frac{5}{3}\ln|x+2|+C$ 3. 3 4.(a) $4\pi[e^e(e-1)-\frac{e^2}{4}$