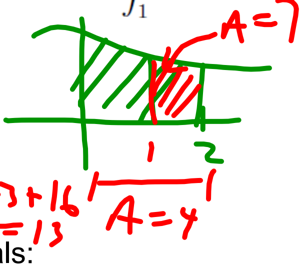
Warm Up...

Binghamton University: Calc 2 Final Exam

1.You are told the following about a function f that is defined on $[0,3]$:

$$\int_0^2 f(x)\,dx = 4, \qquad \int_1^2 f(x)\,dx = 7, \qquad \int_1^3 f(x)\,dx = 16.$$

(i) What is $\int_0^1 f(x)\,dx$? $\Longrightarrow$



$\int_0^1 f(x) = \int_0^2 - \int_1^2$

$= 4 - 7$

$= -3$

(ii) What is $\int_0^3 f(x)\,dx$?

$= \int_0^1 + \int_1^3 = -3+16$
$= 13$

2. Evaluate each of the following integrals:

$$\int (x-1)\sqrt{x^2-2x-3}\;dx \qquad \int \frac{1}{x^3+x}\,dx \qquad \int x \sec x \tan x\;dx$$

3. (20 points) Evaluate $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

$$\int (x-1)\sqrt{x^2-2x-3}\,dx$$

$$u = x^2-2x-3$$

$$du = (2x-2)dx$$

$$\frac{du}{2} = (x-1)dx$$

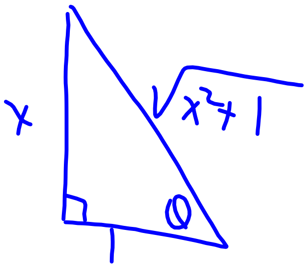
$$\frac{1}{2}\int u^{1/2}du$$

$$\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)+C$$

$$= \frac{1}{3}\left(x^2-2x-3\right)^{3/2}+C$$

$$\int \frac{1}{x^3+x}\,dx$$

$$\int \frac{dx}{x(x^2+1)}$$



$$\int \frac{\cancel{\sec^2\theta}\,d\theta}{\tan\theta\,\cancel{\sec^2\theta}}$$

$\sec\theta=\sqrt{x^2+1}$

$\sec^2\theta=x^2+1$

$\tan\theta=x$

$\sec^2\theta\,d\theta=dx$

$$\int \cot\theta\,d\theta$$

$$\int \frac{\cos\theta}{\sin\theta}\,d\theta$$

$$\ln|\sin\theta|+C$$

$$=\ln\left|\frac{x}{\sqrt{x^2+1}}\right|+C$$

$\ln x-\ln(x^2+1)^{\frac{1}{2}}$

$\ln x-\frac{1}{2}\ln(x^2+1)$

$$\frac{A}{x}+\frac{Bx+C}{x^2+1}=\frac{1}{x(x^2+1)}$$

$Ax^2+A+Bx^2+Cx=1$

$A+B=0$ $A=1$ $C=0$

$B=-1$

$$\int \frac{1}{x}\,dx-\frac{1}{2}\int\frac{2x}{x^2+1}\,dx$$

$$\ln x-\frac{1}{2}\ln(x^2+1)+C$$

$$\int x\underbrace{\sec x\tan x\ dx}_{}$$

$u = x$ $\qquad dv = \sec x \tan x\, dx$

$du = dx$ $\qquad v = \sec x$

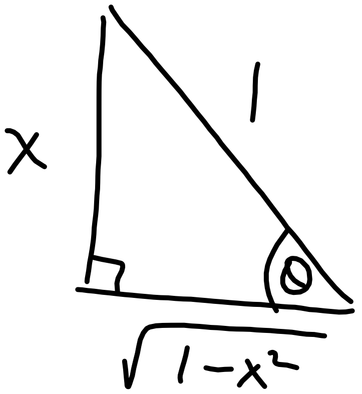
$$= x\sec x - \int \sec x\, dx$$

$$= x\sec x - \int \sec x\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx$$

$$- \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

$$= x\sec x - \ln\left|\sec x + \tan x\right| + C$$

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx$$



$$\int \frac{\sin^3\theta\,\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}$$

$\cos\theta = \sqrt{1-x^2}$

$\sin\theta = x$

$\cos\theta\,d\theta = dx$

$$\int \sin^3\theta\,d\theta$$

$$\int \sin^2\theta\,\sin\theta\,d\theta$$

$$\int (1-\cos^2\theta)\sin\theta\,d\theta$$

$$\int \sin\theta\,d\theta + \int (\cos\theta)^2 -\sin\theta\,d\theta$$

$$= \cos\theta + \frac{1}{3}\cos^3\theta + C$$

$$= -\sqrt{1-x^2} + \frac{1}{3}\left(\sqrt{1-x^2}\right)^3 + C$$

(c) $\int \frac{\tan^{-1}x}{x^2}\,dx$

$u = \tan^{-1}x \qquad dv = x^{-2}\,dx$

$du = \frac{1}{x^2+1}\,dx \qquad v = -x^{-1}$

$= -x^{-1}\tan^{-1}x + \boxed{\int \frac{1}{x(x^2+1)}\,dx}$

Either Partial Fractions or Trig. Sub...

⇒ See warm-up

Set A:

(f) $\frac{A}{x-2} + \frac{Bx+C}{x^2+9}$

Set B

(d) $\int \frac{\sec^4\sqrt{x}\,\tan^2\sqrt{x}}{\sqrt{x}}\,dx$ $\qquad x^{-\frac{1}{2}}$

$$\int \sec^4\sqrt{x}\,\tan^2\sqrt{x}\,(x^{-1/2})\,dx$$

$$2\int \sec^2\sqrt{x}\,\tan^2\sqrt{x}\,\underbrace{\sec^2\sqrt{x}\,\tfrac{1}{2}(x^{-1/2})}_{du}\,dx$$

$$\int (1+\tan^2\sqrt{x})\,\tan^2\sqrt{x}\,(\sec^2\sqrt{x})\,x^{-\frac{1}{2}}$$

$$2\int \overset{u^n}{\tan^2\sqrt{x}}\,\overset{\cdot\, du}{\sec^2\sqrt{x}\,\tfrac{1}{2}x^{-\frac{1}{2}}dx} + 2\int (\tan\sqrt{x})^4\,\sec^2\sqrt{x}\,\tfrac{1}{2}x^{-\frac{1}{2}}\,dx$$

$$\frac{2}{3}\tan^3\sqrt{x} + \frac{2}{5}\tan^5\sqrt{x} + C$$

Set A

(g) $\int \underbrace{16x}\,\underbrace{\tan^{-1}4x\,dx}$

Product

$u = \tan^{-1}4x$

$du = \frac{4}{16x^2+1}dx$

$dv = 16x\,dx$

$v = 8x^2$

$= 8x^2\tan^{-1}4x - \int \frac{32x^2}{16x^2+1}dx$

$32\int \frac{x^2}{16x^2+1}dx$

$$16x^2+1\overset{\frac{1}{16}}{\overline{\smash{)}x^2+0}}$$

$x^2 + \frac{1}{16}$

$-\frac{1}{16}$

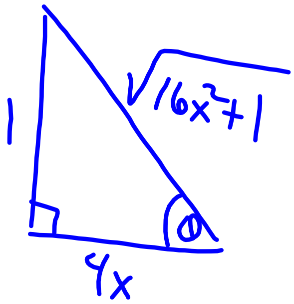
$\frac{1}{16} + \frac{-\frac{1}{16}}{16x^2+1}$

$32\left(\frac{1}{16}\right)\int\left(1 - \frac{1}{16x^2+1}\right)dx$

$2\int dx + \frac{1}{2}\int \frac{\cancel{\csc^2\theta}\,d\theta}{\cancel{\csc^2\theta}}$

$2x + \frac{1}{2}\theta$

$\underbrace{2x + \frac{1}{2}\cot^{-1}4x}$



$\cot\theta = 4x$

$\frac{1}{4}\cot\theta = x$

$-\frac{1}{4}\csc^2\theta\,d\theta = dx$

$\csc\theta = \sqrt{16x^2+1}$

$\csc^2\theta = 16x^2+1$

$8x^2\tan^{-1}4x - \left(2x + \frac{1}{2}\cot^{-1}4x\right) + C$

$$\frac{1}{2}\int \cos 2\theta^{(2)} d\theta + \int 1\, d\theta$$

$$0 \sin 2\theta + \theta$$