Practice Problems...

Evaluate each of the following:

$$\int \tan^3 x \sec^4 x \, dx$$

$$\int \frac{x+4}{x^2+4} \, dx$$

$$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} \, dx$$ (UNB: 2005)

$$\int \frac{\sqrt{2-x^2}}{x} \, dx$$ (UNB: 2004)

$$\int (\csc^2 t \cot^2 t) \, dt$$ (UNB: 2004)

$$\int \frac{\ln(\ln x)}{x} dx$$ (StFX: 2005)

$$\int \tan^3 x \sec^4 x \, dx$$

$$\int \tan^3 x \sec^2 x \sec^2 x \, dx$$

$$\int \tan^3 x (\tan^2 x + 1) \sec^2 x \, dx$$

$$\int (\underbrace{\tan^5 x \sec^2 x}_{u^n \cdot du} + \underbrace{\tan^3 x \sec^2 x}_{u^n \cdot du}) dx$$

$$= \frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + C$$

$$\int \frac{x+4}{x^2+4}\,dx$$

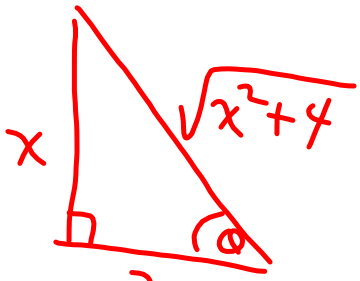
$$\frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$$

$$+4\int \frac{2\sec^2\theta\,d\theta}{4\sec^2\theta}$$

$$+2\int d\theta$$

$$+2\theta$$

$$\frac{1}{2}\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right)+C$$



$$(2\sec\theta)^2 = \left(\sqrt{x^2+4}\right)^2$$

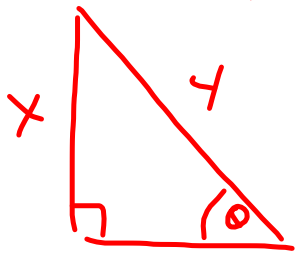
$$4\sec^2\theta = x^2+4$$

$$2\tan\theta = x$$

$$2\sec^2\theta\,d\theta = dx$$

$$\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx$$ (UNB: 2005)

$\left(\sqrt{16-x^2}\right)^3$



$\sqrt{16-x^2}$

$\int \frac{4\cos\theta\,d\theta}{64\cos^3\theta}$

$\frac{1}{16}\int \frac{1}{\cos^2\theta}\,d\theta$

$\frac{1}{16}\int \sec^2\theta\,d\theta$

$\frac{1}{16}\tan\theta + C$

$\frac{1}{16}\left(\frac{x}{\sqrt{16-x^2}}\right) + C$

$(4\cos\theta)^3 = \left(\sqrt{16-x^2}\right)^3$

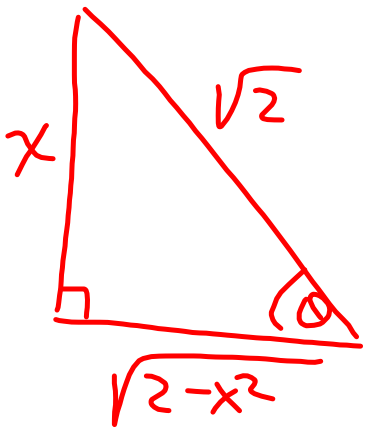
$64\cos^3\theta = (16-x^2)^{3/2}$

$\sin\theta = \frac{x}{4}$

$4\sin\theta = x$

$4\cos\theta\,d\theta = dx$

$$\int \frac{\sqrt{2-x^2}}{x}\,dx$$ (UNB: 2004)



$$\int \frac{\sqrt{2}\cos\theta\,(\sqrt{2}\cos\theta\,d\theta)}{\sqrt{2}\sin\theta}$$

$$\sqrt{2}\int \frac{\cos^2\theta}{\sin\theta}\,d\theta$$

$$\sqrt{2}\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$$

$$\sqrt{2}\int \frac{1}{\sin\theta}\,d\theta - \sqrt{2}\int \sin\theta\,d\theta$$

$$\sqrt{2}\int \csc\theta\,d\theta \left(\frac{\csc\theta-\cot\theta}{\csc\theta-\cot\theta}\right)$$

$$\sqrt{2}\int \frac{\csc^2\theta-\csc\theta\cot\theta}{\csc\theta-\cot\theta}\,d\theta$$

$$\sqrt{2}\ln|\csc\theta-\cot\theta| + \sqrt{2}\cos\theta + C$$

$$\sqrt{2}\ln\left|\frac{\sqrt{2}}{x}-\frac{\sqrt{2-x^2}}{x}\right| + \sqrt{2}\left(\frac{\sqrt{2-x^2}}{\sqrt{2}}\right) + C$$

$\sin\theta = \frac{x}{\sqrt{2}}$

$\sqrt{2}\sin\theta = x$

$\sqrt{2}\cos\theta\,d\theta = dx$

$\cos\theta = \frac{\sqrt{2-x^2}}{\sqrt{2}}$

$\sqrt{2}\cos\theta = \sqrt{2-x^2}$

$\sqrt{x^4+16} \neq x^2+4$

$\sqrt{16+9} \neq 4+3$??

$5^2 = \left(\sqrt{x^4+16}\right)^2$

$$\int (\csc^2 t \cot^2 t)\, dt$$ (UNB: 2004)

u^n du

$$-\int (\cot t)^2 (-\csc^2 t)$$

$$= -\frac{\cot^3 t}{3} + C$$

$$\int \frac{\ln(\ln x)}{x}\,dx$$ (StFX: 2005)

$w=u$ → $uv - \int v\,du$

→ $wv - \int v\,dw$

$u = \ln x$

$du = \frac{1}{x}dx$

$$\int \ln u\,du$$

$w = \ln u \quad dv = du$

$dw = \frac{1}{u}du \quad v = u$

$u = \ln x$

$$= u\ln u - \int u\left(\frac{1}{u}du\right)$$

$$= u\ln u - \int du$$

$$= u\ln u - u + C$$

$$= \ln x \ln(\ln x) - \ln x + C$$