Solve a Trigonometric Equation in Radians

Determine the ⟨general solutions⟩ for the trigonometric equation
$16 = 6 \cos\left(\frac{\pi}{6}\right)x + 14$. Express your answers to the nearest hundredth.

$2 = 6\cos\left(\frac{\pi}{6}x\right)$
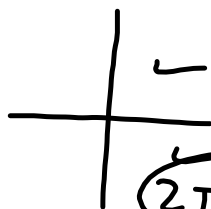
$\frac{1}{3} = \cos\left(\frac{\pi}{6}x\right)$

$\cos^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{6}x$

$2\pi - \theta \Rightarrow$ Q4

$2\pi - 1.23$
$= 5.05$

Q1

$1.23 = \frac{\pi}{6}x$

$\frac{6(1.23)}{\pi} = x$

$2.35 = x$

Q4

$5.05 = \frac{\pi}{6}x$

$x = \frac{6(5.05)}{\pi}$

$x = 9.65$

Extra Step!! ... Calculate Period

$\frac{2\pi}{\pi} = Per$

$\frac{2\pi}{\left(\frac{\pi}{6}\right)} = Per$

$2\pi \times \frac{6}{\pi} = 12$

General Solutions ...

$x = 2.35 + 12K, \; K \in I$

$x = 9.65 + 12K, \; K \in I$

Model Electric Power

The electricity coming from power plants into your house is alternating current (AC). This means that the direction of current flowing in a circuit is constantly switching back and forth. In Canada, the current makes 60 complete cycles each second.

The voltage can be modelled as a function of time using the sine function $V = 170 \sin 120\pi t$.

a) What is the period of the current in Canada?

b) Graph the voltage function over two cycles. Explain what the scales on the axes represent.

c) Suppose you want to switch on a heat lamp for an outdoor patio. If the heat lamp requires 110 V to start up, determine the time required for the voltage to first reach 110 V.

Did You Know?

The number of cycles per second of a periodic phenomenon is called the frequency. The hertz (Hz) is the SI unit of frequency. In Canada, the frequency standard for AC is 60 Hz.
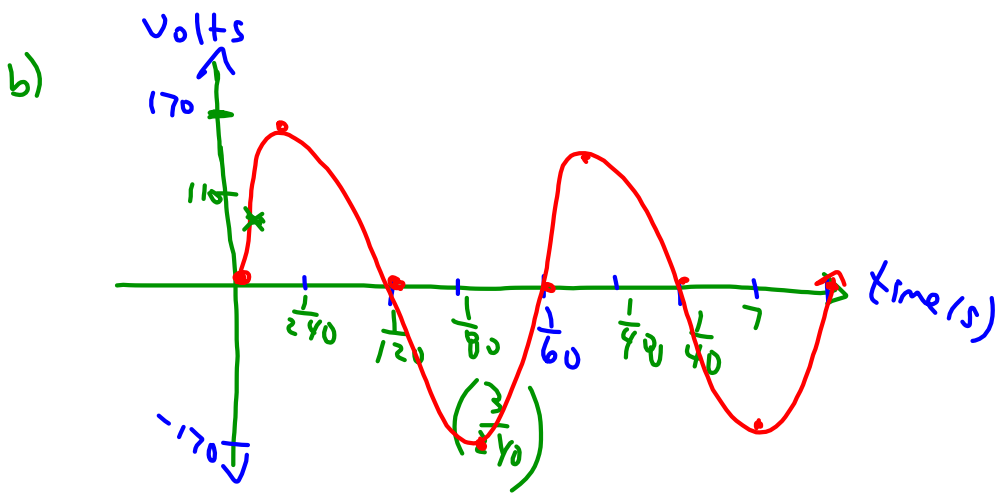
Voltages are expressed as root mean square (RMS) voltage. RMS is the square root of the mean of the squares of the values. The RMS voltage is given by $\frac{\text{peak voltage}}{\sqrt{2}}$. What is the RMS voltage for Canada?

(a) $Per = \dfrac{2\pi}{k}$

$Per = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ sec

b)

c) $110 = 170 \sin(120\pi t)$

$\dfrac{110}{170} = \sin 120\pi t$

$\sin^{-1}\left(\dfrac{110}{170}\right) = 120\pi t$

$0.7037 = 120\pi t$

$t = \underline{0.00187 \text{ sec}}$

Q2

Solve the following trigonometric equations:

$$-2.8 \sin\left(\frac{\pi}{6}(x - 12)\right) + 16 = 17 ,$$

$$-4\pi \le x \le 4\pi$$

(circled, green)
6.7
11.3
-5.3
-0.7

$$-12.6 \le x \le 12.6$$

$$\frac{1}{-2.8} = \sin\frac{\pi}{6}(x-12)$$

$$\sin^{-1}(-0.3571) = \frac{\pi}{6}(x-12) \quad \text{Q3,4}$$

$$(\text{Ref} \neq 0.3652$$

Q3

$$\pi + 0.3652 = \frac{\pi}{6}(x-12)$$

$$3.5068 = \frac{\pi}{6}(x-12)$$

$$\frac{6(3.5068)}{\pi} + 12 = x$$

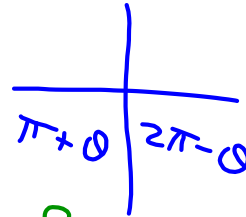$$x = 18.6975 - \text{Period} (12)$$

$$\boxed{x = 6.7}$$

$$\frac{\pi + \theta \mid 2\pi - \theta}{}$$

Need the Period...

$$\frac{2\pi}{\left(\frac{\pi}{6}\right)} = \text{Per}$$

$$2\pi \times \frac{6}{\pi} = 12$$

Q4

$$2\pi - 0.3652 = \frac{\pi}{6}(x-12)$$

$$5.91798 = \frac{\pi}{6}(x-12)$$

$$\frac{6 \times 5.9}{\pi} + 12 = x$$

$$x = 23.3 \quad \text{Not between } -4\pi \ \& \ 4\pi$$

$$\frac{-12}{\boxed{11.3}}$$

Other Possibilities...

$$6.7 - 12 = \boxed{-5.3}$$

$$11.3 - 12 = \boxed{-0.7}$$

$6.2 \sin (4(x + 8°)) - 1 = 4,$

$0° \leq x \leq 360°$

$\sin^{-1}\left(\dfrac{5}{6.2}\right) = \sin(4(x+8))$

$\dfrac{180-\theta \mid \theta}{}$ $\dfrac{180-\theta \mid \theta}{}$

$\underline{Q1}$

$53.75 = 4(x+8)$

$X = 5.4$

$= 95.4$
$= 185.4$
$= 275.4$

$Per = \dfrac{360}{K} = \dfrac{360}{4}$
$= 90$

$\underline{Q2}$

$126.25 = 4(x+8)$

$\dfrac{126.25}{4} - 8 = X$

$X = 23.6$

$= 113.6$
$= 203.6$
$= 293.6$

One of the main food sources for the Arctic fox is the lemming. Suppose the population, L, of lemmings in the region is modelled by the function

$$L(t) = 5000 \sin \frac{\pi}{12}(t - 12) + 10\,000.$$

(a) Determine the lemming population during year 15 of this study.

(b) Determine the **second** instance that the lemming population reached 8000.

<u>REVIEW - Sketching Trigonometric Functions</u>

- sinusoidal functions
    - properties: domain/range, amplitude, period, phase shift, vertical translation, eq'n of sinusoidal axis, mapping notation.
    - sketching equation in standard form.
- finding the function (both a sine/cosine) given a graph
- solving trigonometric equations where period is not 360
- applications of sinusoidal functions.
    - sketch
    - develop a function
    - use function to answer question
- sketches of all SIX trigonometric ratios

Textbook Review....

Pg. 282 - 285

#4, 6, 7, 8, 10, 11, 20, 21, 22, 23, 24

Practice Test: Page 286 - 287
#1 - 7
#11, 12, 14, 15, 16